Cognitive Interference Channels

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Interference channel (IFC)

- Multiple users $\rightarrow$ interference
- Carleial '75: ”cases where interference does not reduce capacity” $\rightarrow$ it's decodable (as opposed to noise)
- How do we deal with interference?
Interference channel (IFC)

- Han-Kobayashi '81: largest known achievable region
Interference channel (IFC)

- ElGamal-Costa '82: capacity of certain deterministic IFCs
  \[ Y_1 = X_1 + g_1(X_2) \rightarrow X_{2,\text{common}} = g_1(X_2) \]
  \[ Y_1 = g_2(X_1) + X_2 \rightarrow X_{1,\text{common}} = g_2(X_1) \]

- Etkin-Tse-Wang '07: capacity of Gaussian IFCs to within 1 bit/sec/Hz
  \[ Y_1 = X_1 + a X_2 + Z_1 \rightarrow \text{Var}[a X_{2,\text{private}}] \leq \text{Var}[Z_1] \]
  \[ Y_1 = b X_1 + X_2 + Z_2 \rightarrow \text{Var}[b X_{1,\text{private}}] \leq \text{Var}[Z_2] \]
IFC with Generalized Feedback (IFC-GF)

- Sources sense the channel → source cooperation (with Echo Yang)
IFC with conferencing encoders

- GF = error free bit pipes with finite capacity
Cognitive IFC

- Limiting case of conferencing encoders: anti-causal message knowledge

NB: elements of IFC and of BC
Our Contributions

- New outer bound (ITW'09)
- New inner bound (IZS'10)
- Capacity to within 1.07 bits/sec/Hz for the Gaussian channels (ITW'10)
- Capacity results (new):
  - `Better cognition' channels
  - Semi-deterministic channels
  - Certain Gaussian channels
State-of-the-art

- **2005:** Devroye et al
  - Introduces channel model; inner bound.

- **2006:** Wu et al and Jovicic et al
  - BC-type outer bound; capacity in weak interference.

- **2007:** Maric et al
  - Inner and outer bounds; capacity in very strong interference.

- **2008:** Cao et al
  - General inner bound with BC idea.

- **2009:** Jiang et al
  - General inner bound (from IFC-CR).

- **2009:** Rini et al
  - Inner and outer bounds; capacity for certain classes of channels; constant gap for Gaussian channels.
State-of-the-art

- Inner bounds
Available results

- Wu et al outer bound by adapting Körner and Marton's BC outer bound

\[
R_1 \leq I(Y_1; X_1|X_2),
\]

\[
R_2 \leq I(U, X_2; Y_2),
\]

\[
R_1 + R_2 \leq I(U, X_2; Y_2) + I(Y_1; X_1|U, X_2),
\]

\[U = \text{msg from 1 to 2}\]
Capacity results

- **Maric et al** very strong interference

\[ I(X_1; Y_1 | X_2) \leq I(X_1; Y_2 | X_2) + I(Y_1; X_1, X_2) \leq I(Y_2; X_1, X_2) = \text{Capacity!} \]
Capacity results

- **Wu et al** and **Jovivic et al** very weak interference

\[
I(X_2; Y_2) \leq I(X_2; Y_1) + I(U; Y_2|X_2) \leq I(U; Y_1|X_2) = \text{Capacity!}
\]
Our inner bound

primary—knows
$W_2$ only

cognitive—knows
$W_1$ and $W_2$

$U_{1pb}$ and $U_{2pb}$ sent by cognitive!
New inner bound

- Enc 1 (cognitive): DPC of \((U_{1c} U_{1pb})\) against \((U_{2pa} U_{2pb} X_2)\) conditioned on \(U_{2c}\)
- Enc 2 (primary): \(U_{2c} U_{2pa}\)
- Decoding as in IFC
Our outer bound

- Sato's "worst joint, same marginal" idea for non-cooperative receivers

\[
R_1 \leq I(Y_1; X_1|X_2),
\]

\[
R_2 \leq I(X_1, X_2; Y_2),
\]

\[
R_1 + R_2 \leq I(X_1, X_2; Y_2) + I(Y_1; X_1|Y'_2, X_2),
\]

such that \( P_{Y'_2|X_1,x_2} = P_{Y_2|X_1,x_2} \)
Semi-Deterministic channels

- If $Y_1 = f_1(X_1, X_2)$ for cognitive rx:

  \[
  R_1 \leq H(Y_1|X_2) \\
  R_2 \leq I(Y_2; U, X_2) \\
  R_1 + R_2 \leq I(Y_2; U, X_2) + H(Y_1|U, X_2)
  \]

- Achievability: $U_{1c} = U_{2c} = 0$. 
Deterministic channels

- If $Y_2 = f_2(X_1, X_2)$ too:

\[
R_1 \leq H(Y_1 | X_2) \\
R_2 \leq H(Y_2) \\
R_1 + R_2 \leq H(Y_2) + H(Y_1 | X_2, Y_2).
\]

- Achievability: $U_{1pb} = Y_1$ and $U_{2pb} = Y_2$ as for deterministic BC!
High SNR deterministic channels

- Achievability: $U_{1pb} = Y_1$, $U_{2pb} = Y_2$ for all channel parameters (no need to distinguish strong/weak/mixed interference)
High SNR deterministic channels

\[ n_{21} \quad n_{11} \]

\[ n_{22} \]

Noise floor

Noise floor
High SNR deterministic channels
Gaussian channels

- Our outer bound unifies two known bounds (Wu et al very weak interf., and Maric et al very strong interf.)
Gaussian channels

\[ |a| = \text{INR}_1 \]

\[ |b| = \text{INR}_2 \]

primary

weak interference

very strong interference

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Gaussian channels: gap in strong interference

Our outer bound

Straight line approximation of our outer bound

Goal: bound this distance

$R_2$

MISO strategy

Some strategy

$R_1$
Gaussian channels

\[ |a| = \text{INR}_1 \]

primary

\[ |b| = \text{INR}_2 \]

cognitive

\[ \text{Gap} = 1.07 \]

\[ \text{capacity!!!} \]

\[ \text{very strong interference} \]

\[ \text{weak interference} \]
Gap

Achievability:

\[ Y_1 \sim U_{1pb} \]

\[ = X_1 + aX_2 + Z'_1 \]

\[ \approx X_1 + aX_2 \]

\[ \approx X_1 + \frac{P_1}{P_1 + 1} aX_2 \]
Gap
Z-IFC b=3 a=0
Part II: IFC-CR

- Outer bounds
- Achievability for (certain) high SNR deterministic channels

![Diagram with elements labeled as Enc2, Enc1, Encc, DMC, Dec1, Dec2, W1, W2, X^n, Y^n, Y^4_n, Y^3_n]
State-of-the-art: IFC-CR

- 2007: Sahin et al
  - Model and inner bound.

- 2008: Sridharan et al
  - Inner and outer bound for Gaussian sumrate.

- 2009: Jiang et al
  - Inner bound.

- 2010: Rini et al
  - Outer bound, tight for certain high SNR deterministic channels.
Our contributions

- Outer bound for general channels (à la Sato's CSM)
- Tightened outer bound for a class of semi-deterministic channels
- Achievability for high SNR channels for all parameters we tried (not finished yet)
High SNR channel

- Interesting achievable schemes:
  \[ n_{11} > n_{1c} > n_{12} \text{ and } n_{22} > n_{2c} > n_{21} (R_1 < n_{11}, R_2 < n_{22}) \]
On going work

- High SNR channel: prove capacity for all parameter regimes
- (Semi) deterministic channel: capacity for certain classes
- Gaussian channel: compare our outer bound with existing ones (cognitive MIMO)
- Gaussian channel: finite gap
Conclusions

- **C-IFC:**
  - New inner bound, New Sato-type outer bound
  - Gap less 1.07 bits for Gaussian
  - Capacity for certain classes of channels

- **IFC-CR:**
  - New Sato-type outer bound
  - Capacity for high SNR channel for all parameter range we tried
  - Working on Gaussian channel

- **IFC-GF:**
  - New Sato-type outer bound
  - Working on reducing gap for Gaussian channel
  - Working on multiplicative gap
Thank you!!!