Cognitive Interference Channels

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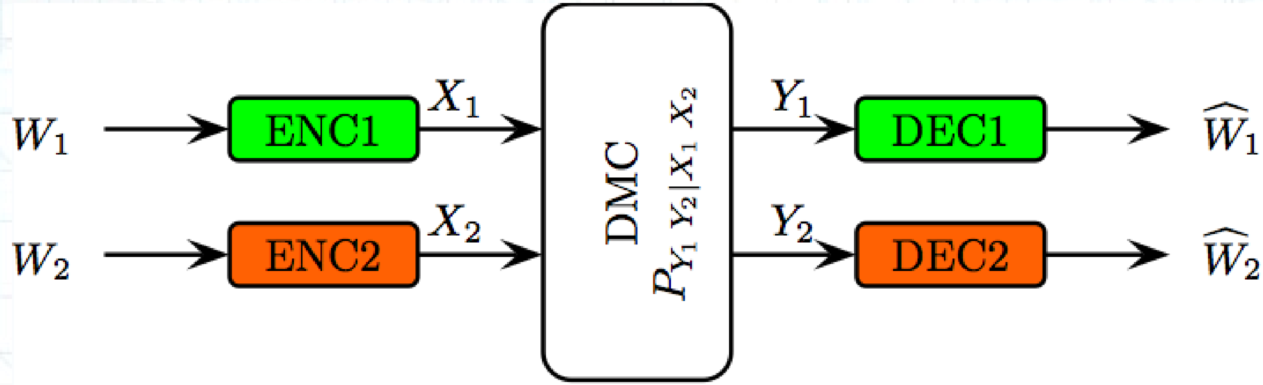
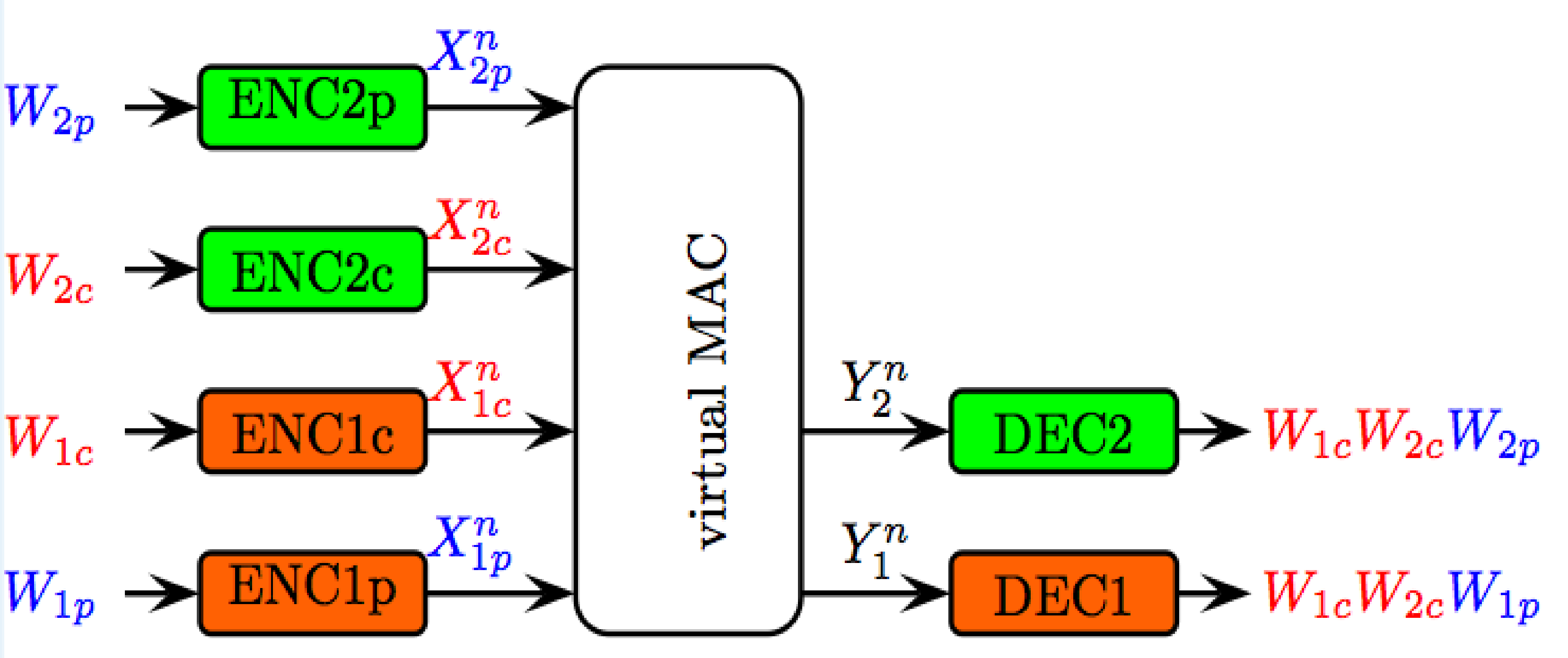
Interference channel (IFC)

- Multiple users → interference
- Carleial '75: "cases where interference does not reduce capacity" → it's decodable (as opposed to noise)
- How do we deal with interference?



Interference channel (IFC)

- Han-Kobayashi '81: largest known achievable region



Interference channel (IFC)

- ElGamal-Costa '82: capacity of certain deterministic IFCs

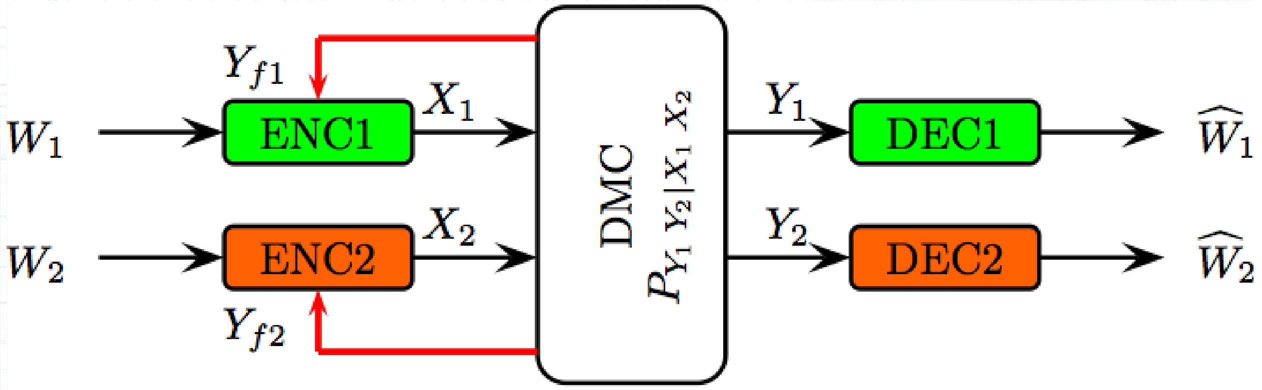
$$Y_1 = X_1 + g_1(X_2) \longrightarrow X_{2,\text{common}} = g_1(X_2)$$
$$Y_1 = g_2(X_1) + X_2 \longrightarrow X_{1,\text{common}} = g_2(X_1)$$

- Etkin-Tse-Wang '07: capacity of Gaussian IFCs to within 1 bit/sec/Hz

$$Y_1 = X_1 + a\,X_2 + Z_1 \longrightarrow \text{Var}[a\,X_{2,\text{private}}] \le \text{Var}[Z_1]$$
$$Y_1 = b\,X_1 + X_2 + Z_2 \longrightarrow \text{Var}[b\,X_{1,\text{private}}] \le \text{Var}[Z_2]$$

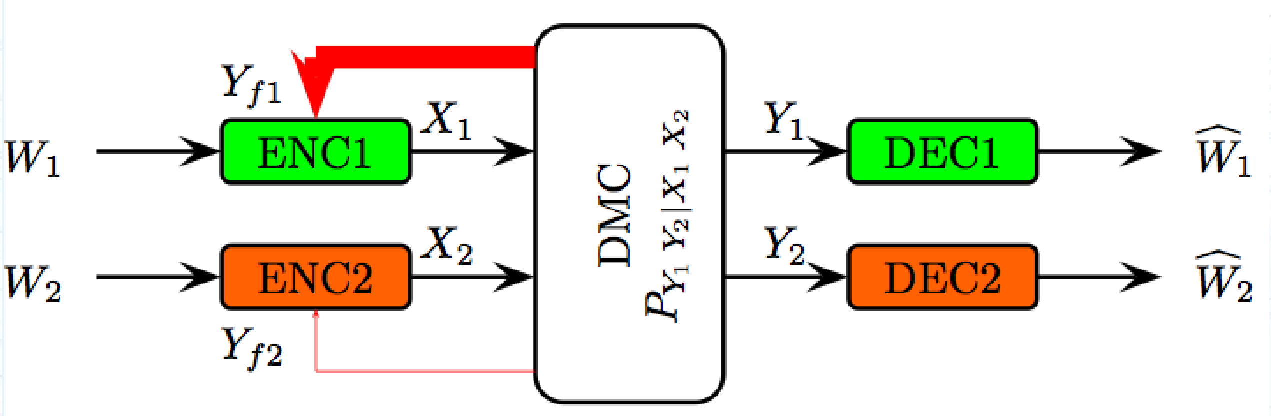
IFC with Generalized Feedback (IFC-GF)

- Sources sense the channel → source cooperation (with Echo Yang)



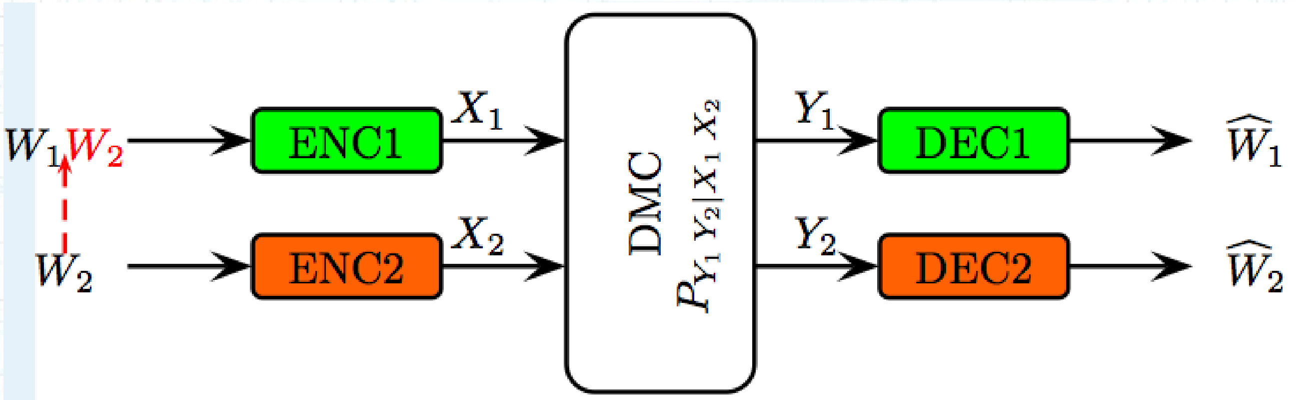
IFC with conferencing encoders

- GF = error free bit pipes with finite capacity



Cognitive IFC

- Limiting case of conferencing encoders: anti-causal message knowledge



NB: elements of IFC and of BC

Our Contributions

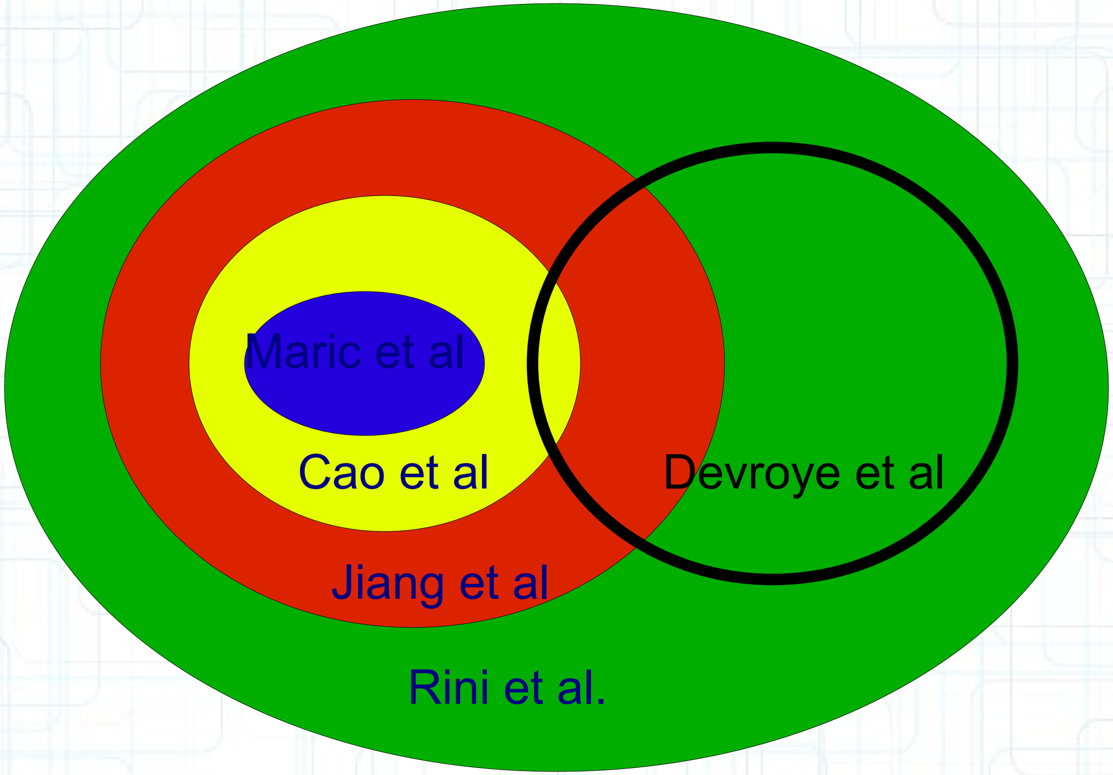
- New outer bound (ITW'09)
- New inner bound (IZS'10)
- Capacity to within 1.07 bits/sec/Hz for the Gaussian channels (ITW'10)
- Capacity results (new):
  - `Better cognition' channels
  - Semi-deterministic channels
  - Certain Gaussian channels

State-of-the-art

- 2005: **Devroye** *et al*
  - Introduces channel model; inner bound.
- 2006: **Wu** *et al* and **Jovicic** *et al*
  - BC-type outer bound; capacity in weak interference.
- 2007: **Maric** *et al*
  - Inner and outer bounds;
    capacity in very strong interference.
- 2008: **Cao** *et al*
  - General inner bound with BC idea.
- 2009: **Jiang** *et al*
  - General inner bound (from IFC-CR).
- 2009: **Rini** *et al*
  - Inner and outer bounds; capacity for certain classes
    channels; constant gap for Gaussian channels.

State-of-the-art

- Inner bounds



Available results

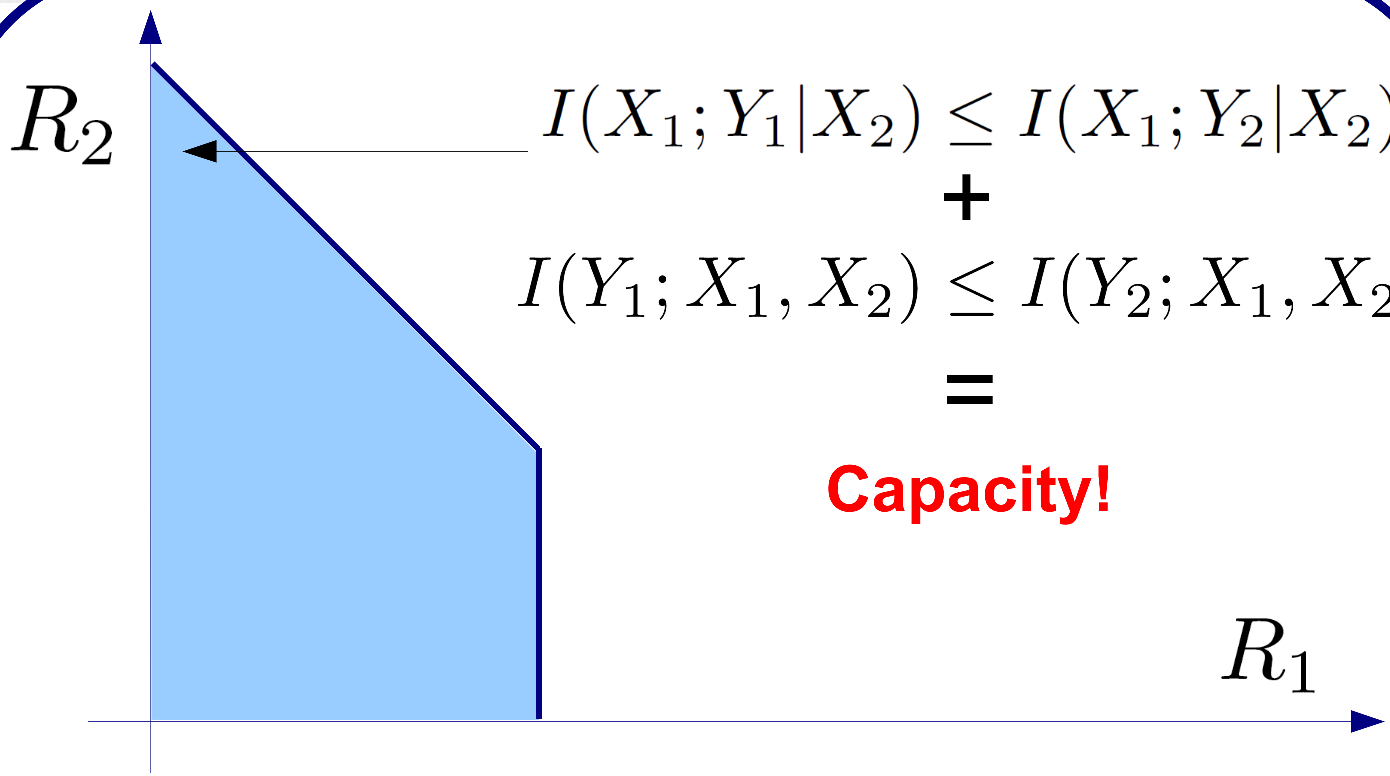
- **<u>Wu</u>** *et al* outer bound by adapting Körner and Marton's BC outer bound

$$\boxed{\begin{aligned} R_1 &\leq I(Y_1; X_1 | X_2), \\ R_2 &\leq I(U, X_2; Y_2), \\ R_1 + R_2 &\leq I(U, X_2; Y_2) + I(Y_1; X_1 | U, X_2), \end{aligned}}$$

U = msg from 1 to 2

Capacity results

- **<u>Maric</u>** *et al* very strong interference



$$I(X_1;Y_1|X_2) \leq I(X_1;Y_2|X_2)$$

+

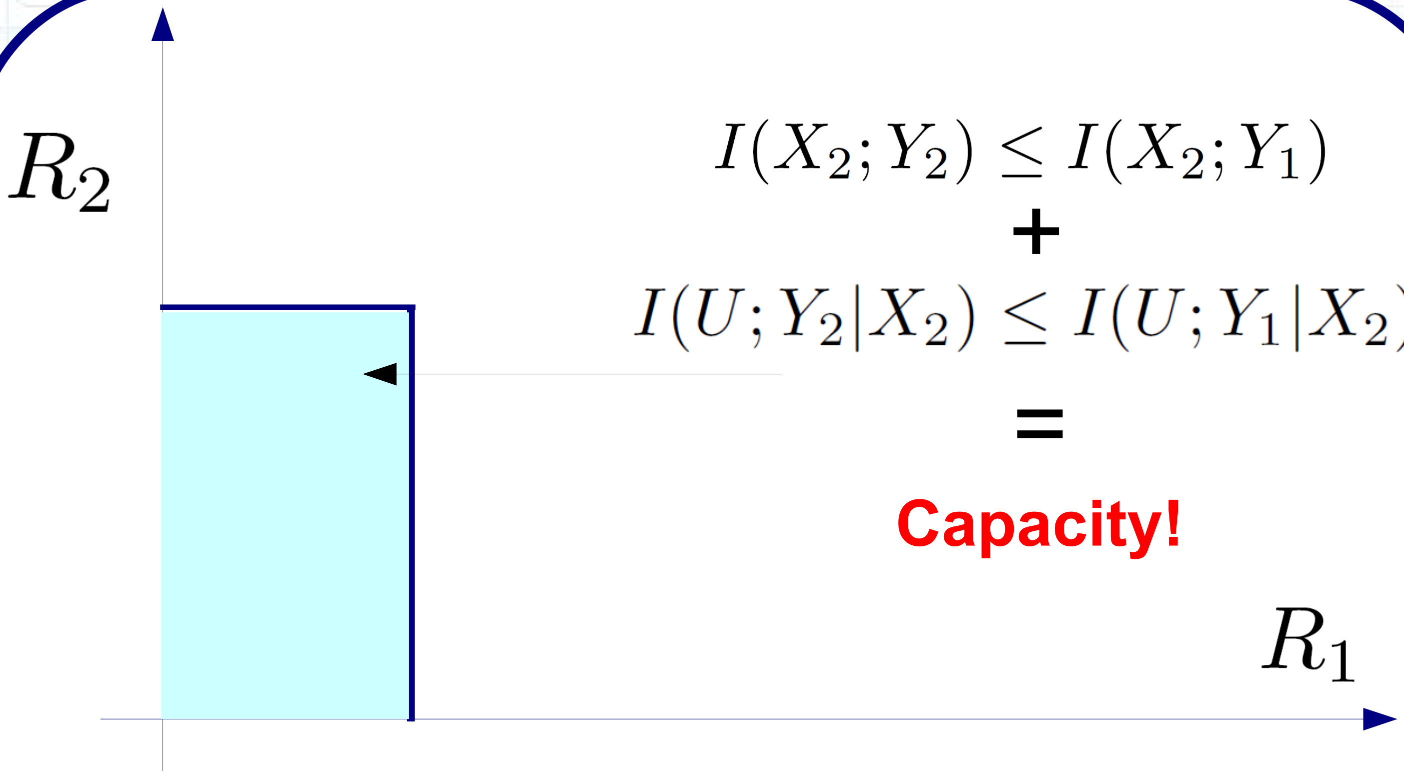
$$I(Y_1;X_1,X_2) \leq I(Y_2;X_1,X_2)$$

=

Capacity!

Capacity results

- **<u>Wu</u>** *et al* and **<u>Jovivic</u>** *at al* very weak interference



$$I(X_2;Y_2) \leq I(X_2;Y_1)$$

+

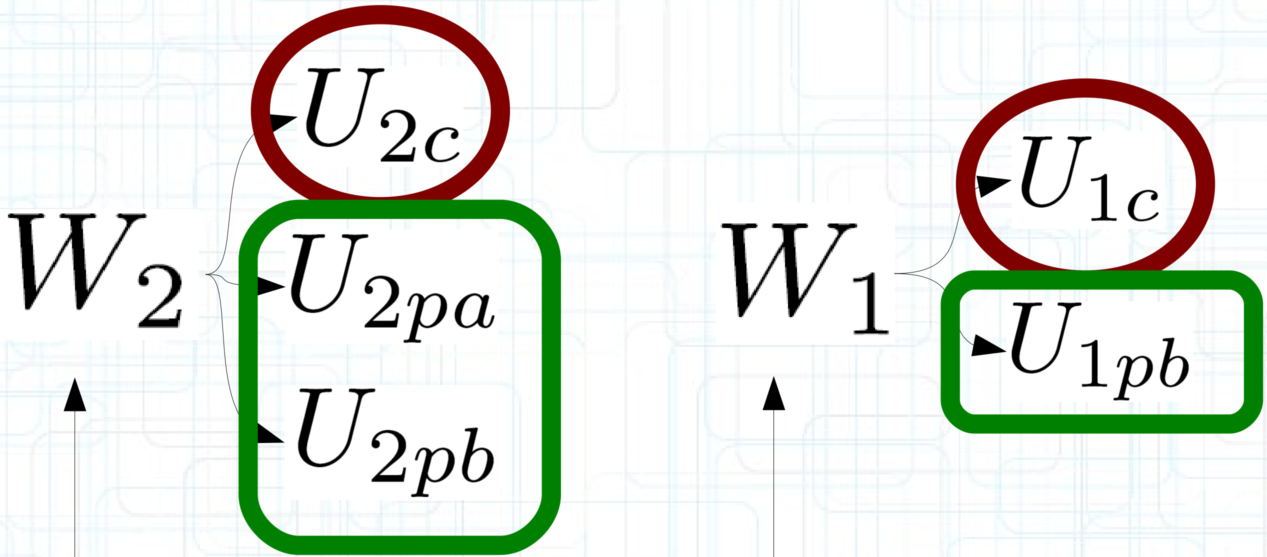
$$I(U;Y_2|X_2) \leq I(U;Y_1|X_2)$$

=

Capacity!

Our inner bound

c = common
p = private
a = alone
b = broadcast

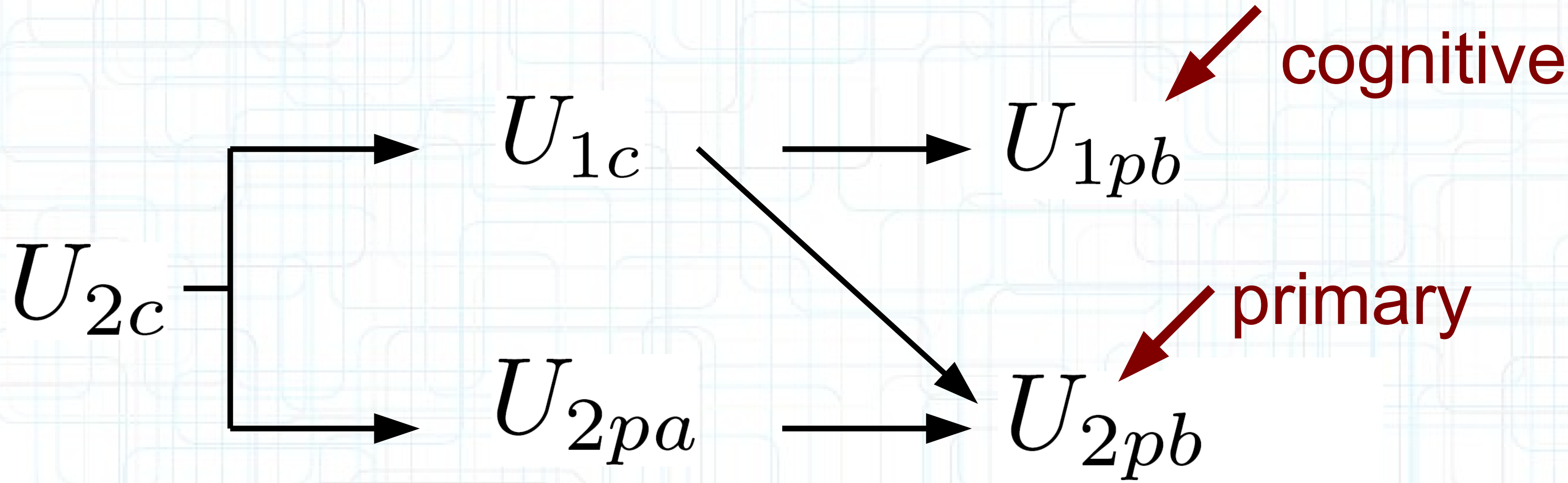


primary—knows W_2 only

cognitive—knows W_1 <u>and</u> W_2

U_{1pb} and U_{2pb} sent by cognitive!

New inner bound



- Enc 1 (cognitive): DPC of (U_{1c} U_{1pb}) against (U_{2pa} U_{2pb} X_2) conditioned on U_{2c}
- Enc 2 (primary): U_{2c} U_{2pa}
- Decoding as in IFC

Our outer bound

- **<u>Sato</u>**'s "*worst joint, same marginal*" idea for non-cooperative receivers

$$\boxed{\begin{aligned} R_1 &\leq I(Y_1; X_1 | X_2), \\ R_2 &\leq I(X_1, X_2; Y_2), \\ R_1 + R_2 &\leq I(X_1, X_2; Y_2) + I(Y_1; X_1 | {Y'}_2, X_2), \end{aligned}}$$

such that $P_{{Y'}_2|X_1,X_2} = P_{Y_2|X_1,X_2}$

Semi-Deterministic channels

- If $Y_1 = f_1(X_1,X_2)$ for cognitive rx:

$$\begin{aligned} R_1 &\leq H(Y_1|X_2) \\ R_2 &\leq I(Y_2;U,X_2) \\ R_1+R_2 &\leq I(Y_2;U,X_2)+H(Y_1|U,X_2) \end{aligned}$$

- Achievability: $U_{1c} = U_{2c} = 0$.

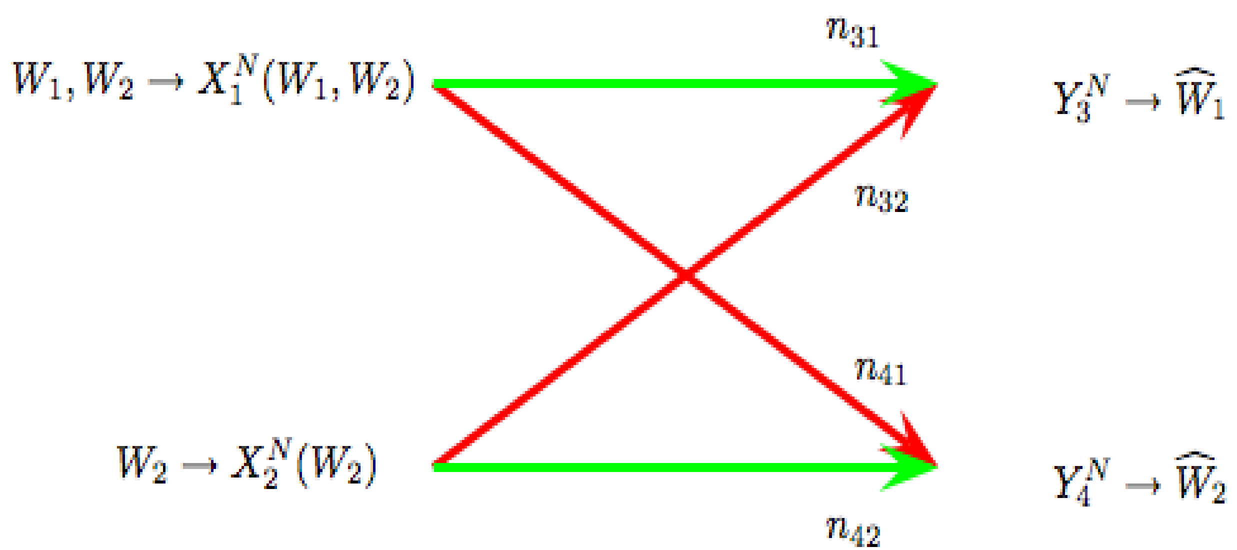
Deterministic channels

- If $Y_2 = f_2(X_1,X_2)$ too:

$$
\boxed{
\begin{aligned}
R_1 &\leq H(Y_1|X_2) \\
R_2 &\leq H(Y_2) \\
R_1 + R_2 &\leq H(Y_2) + H(Y_1|X_2, Y_2).
\end{aligned}
}
$$

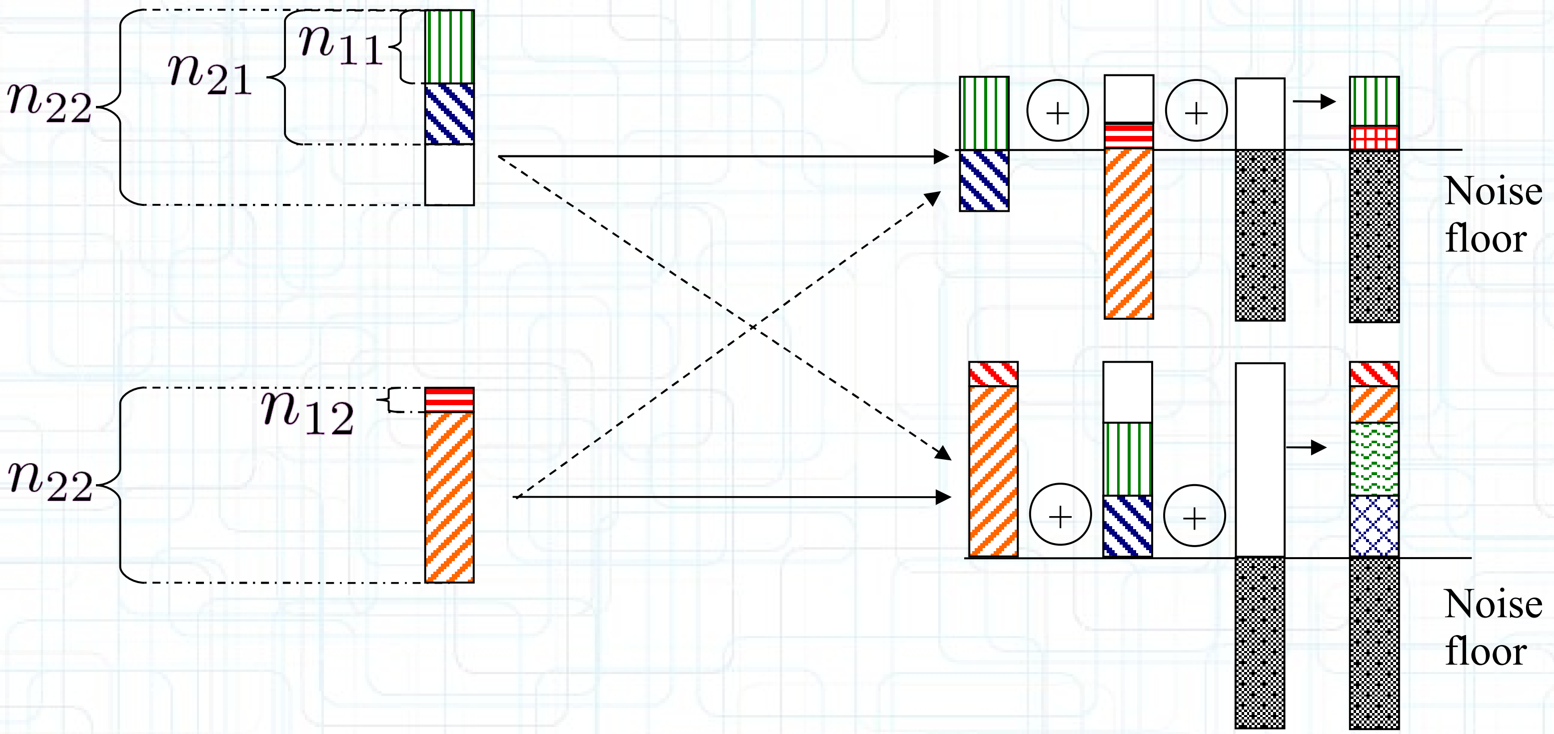
- Achievability: $U_{1pb} = Y_1$ and $U_{2pb} = Y_2$ as for deterministic BC!

High SNR deterministic channels

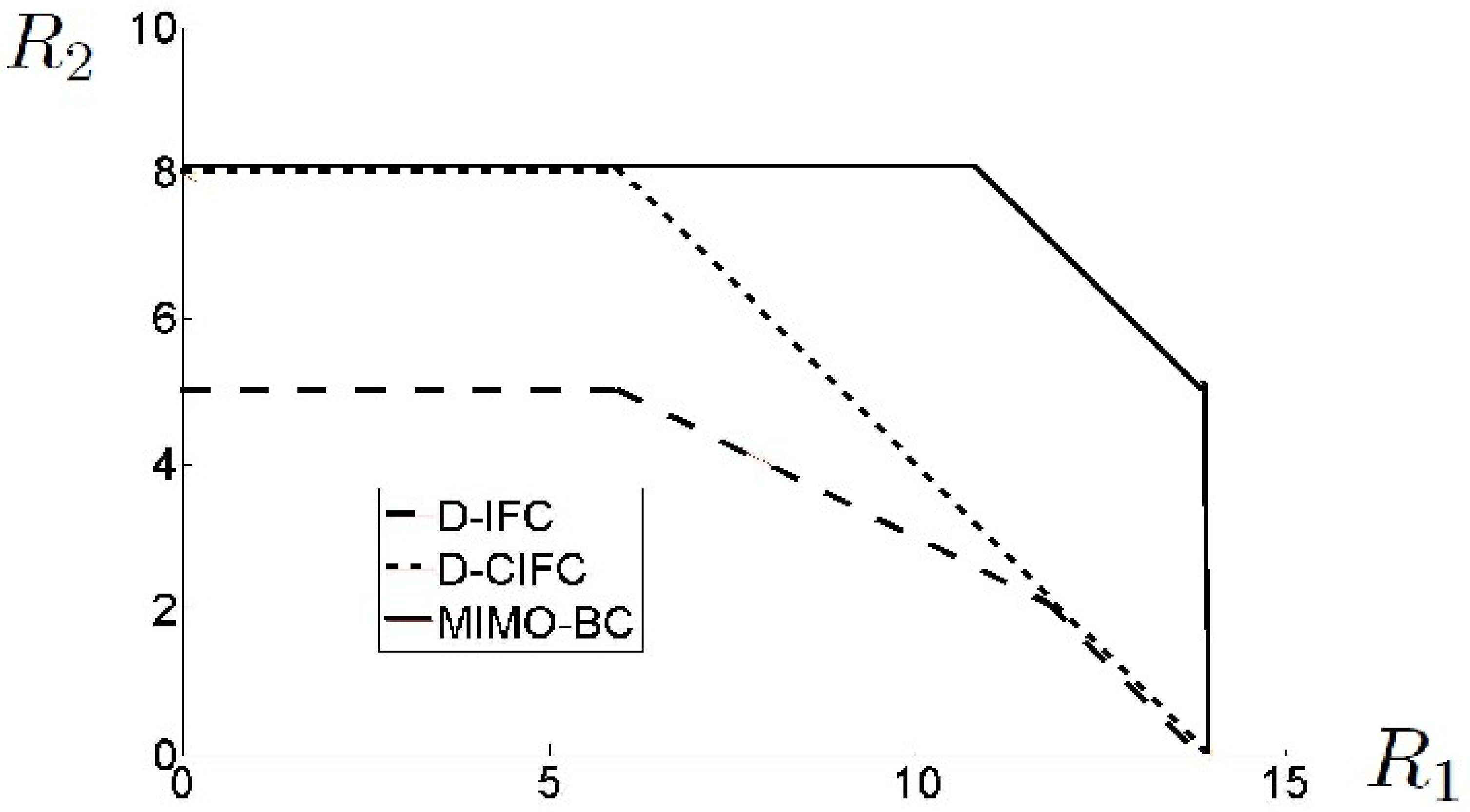


- Achievability: $U_{1pb} = Y_1$, $U_{2pb} = Y_2$ for all channel parameters (no need to distinguish strong/weak/mixed interference)

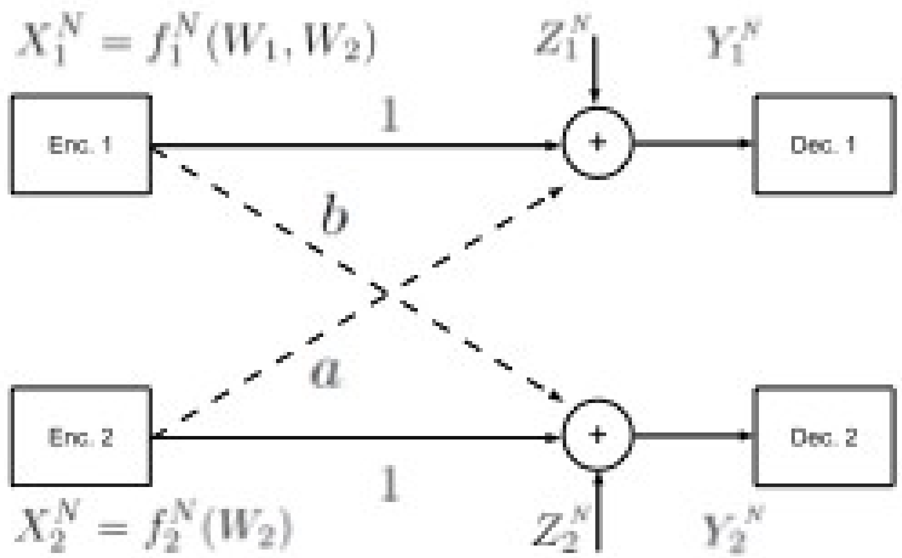
High SNR deterministic channels



High SNR deterministic channels

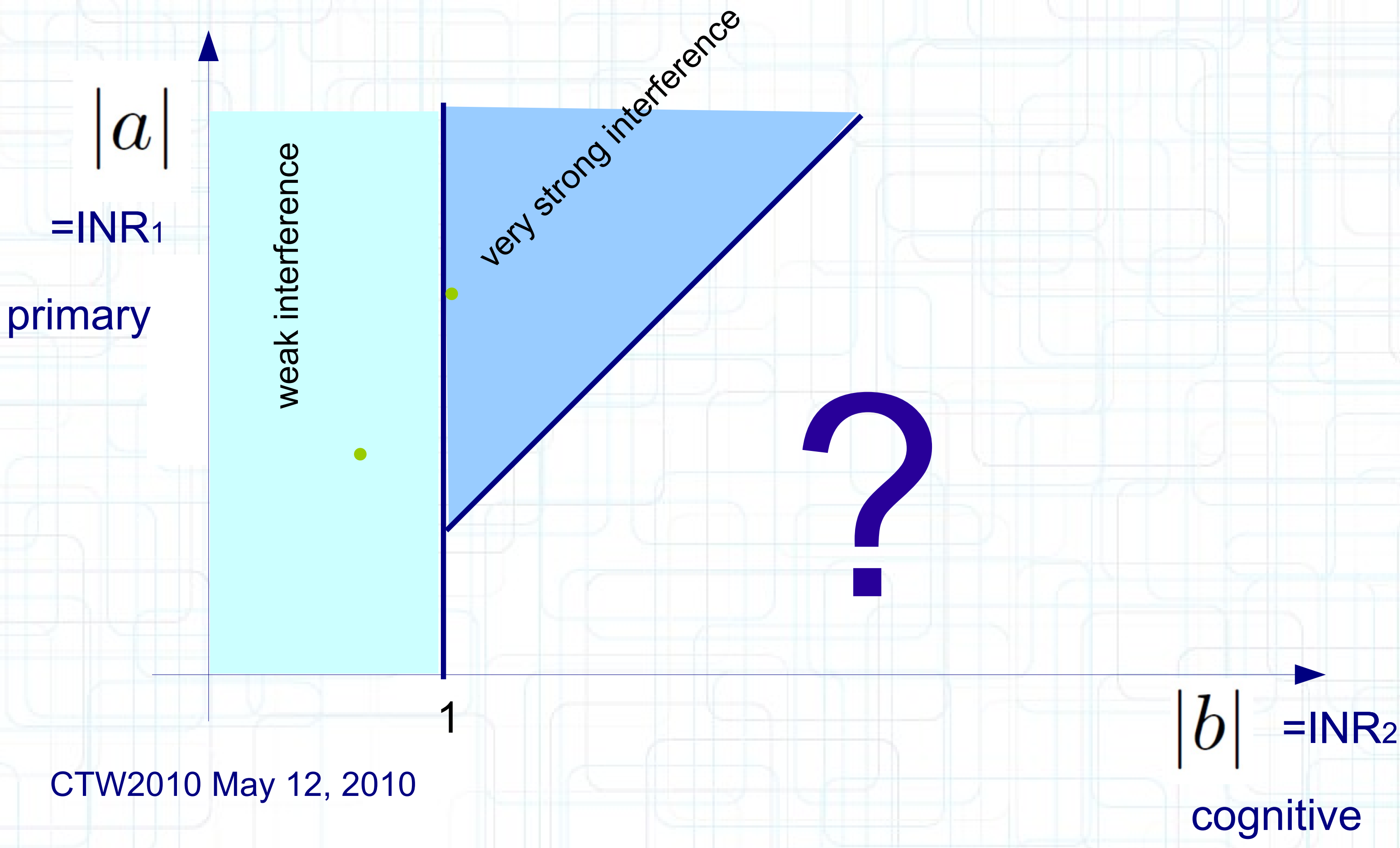


Gaussian channels

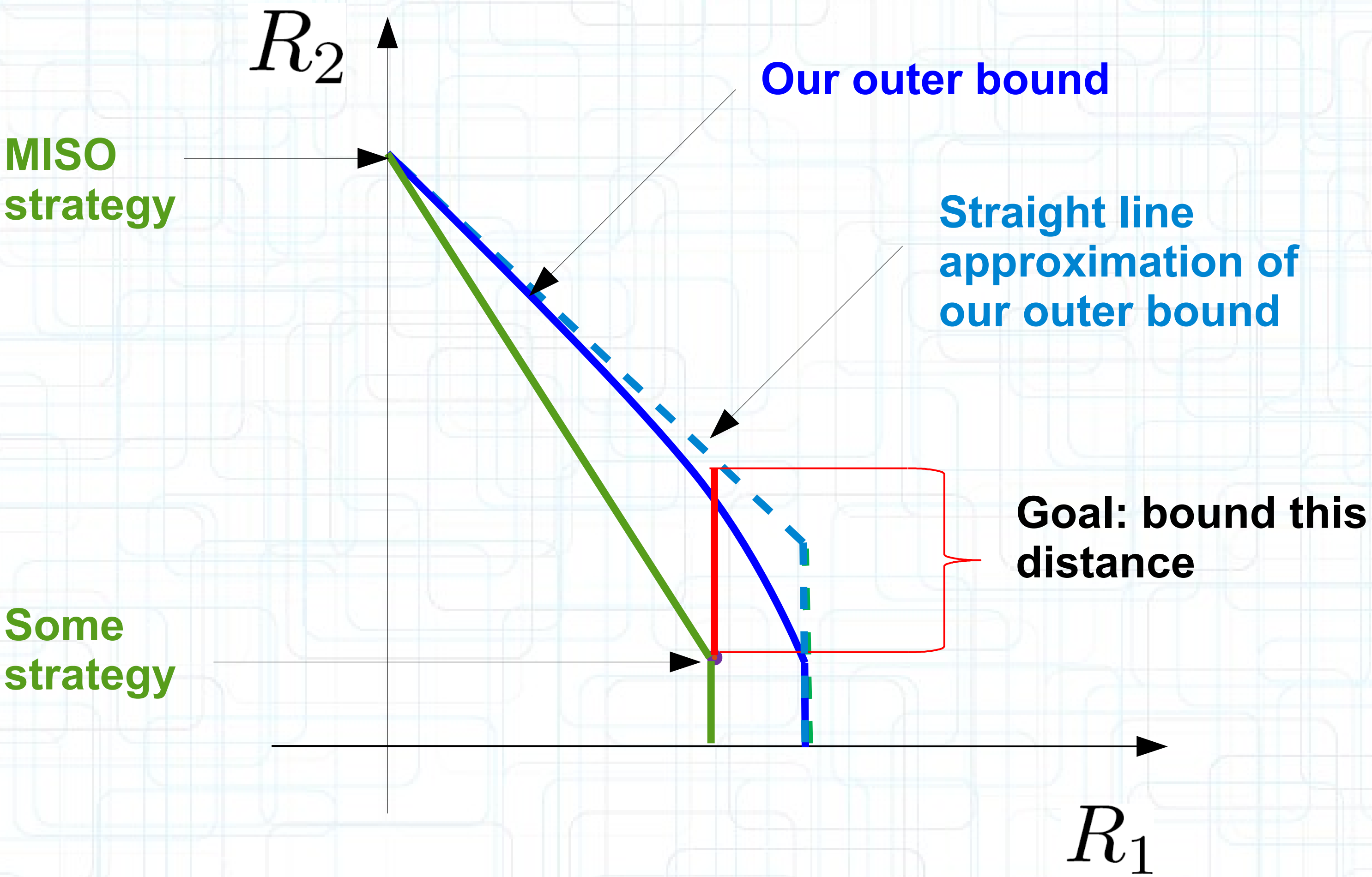


- Our outer bound unifies two know bounds (**<u>Wu</u>** *et al* very weak intef., and **<u>Maric</u>** *et al very strong interf.*)

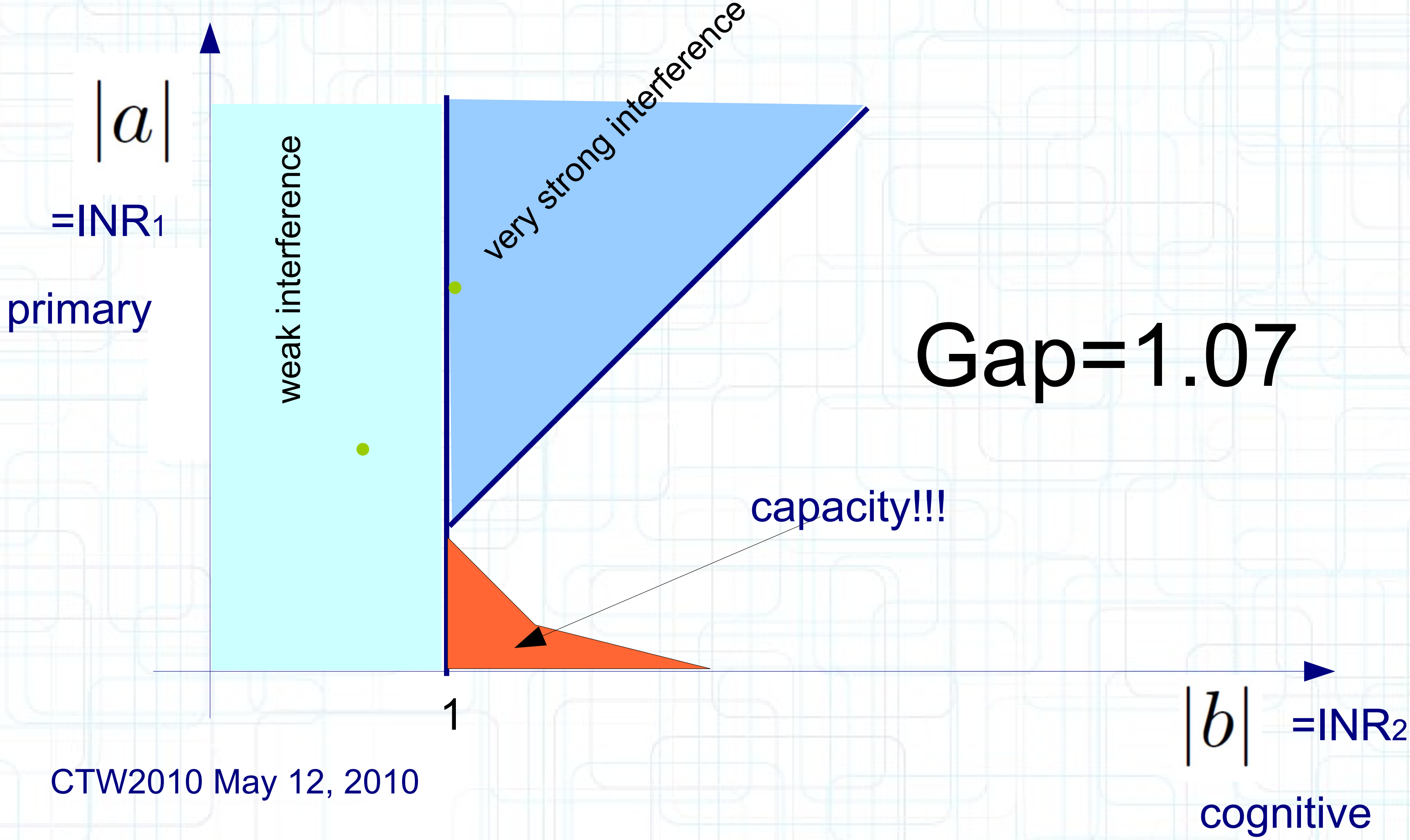
Gaussian channels



Gaussian channels: gap in strong interference



Gaussian channels



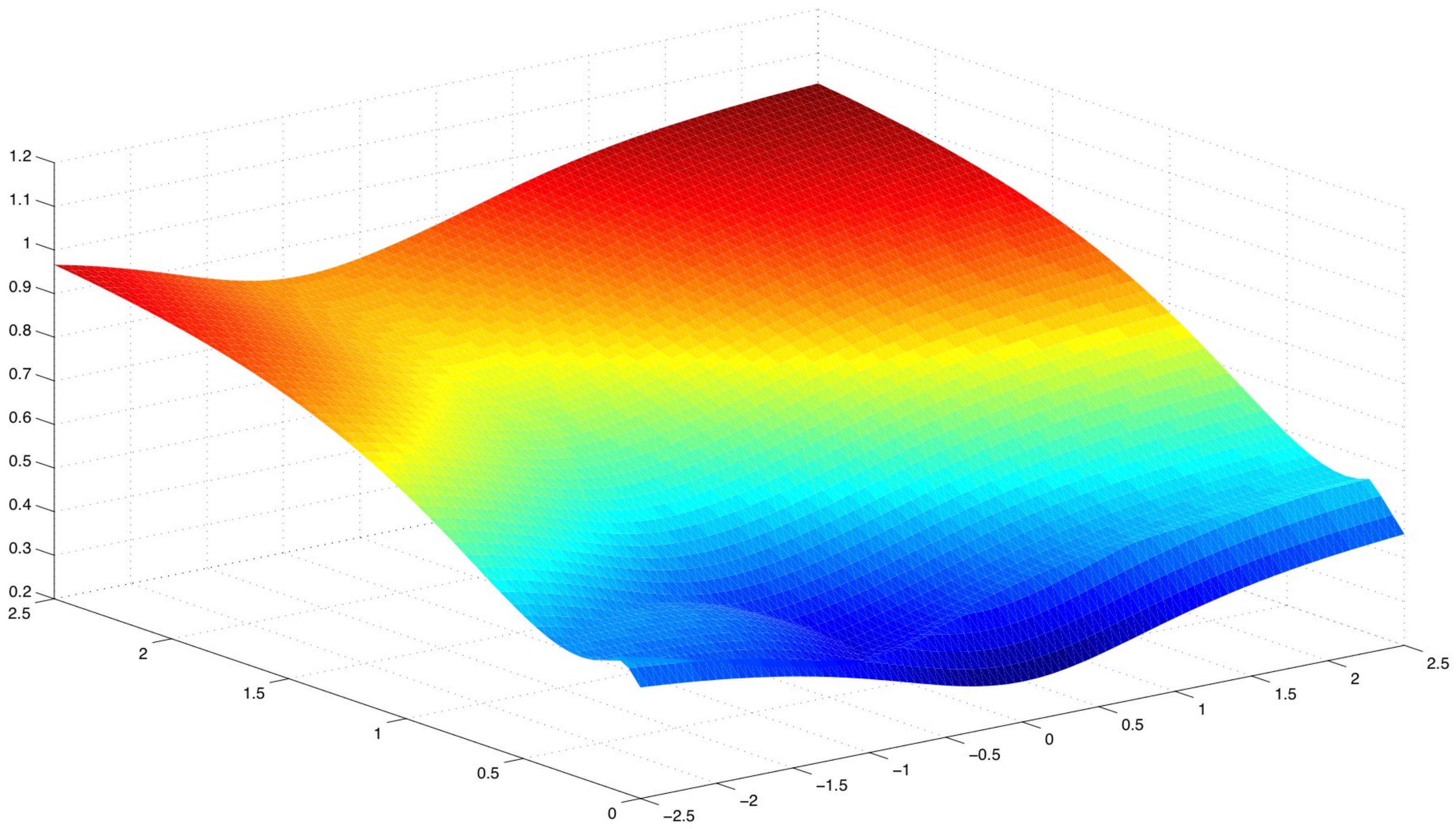
Gap=1.07

Gap

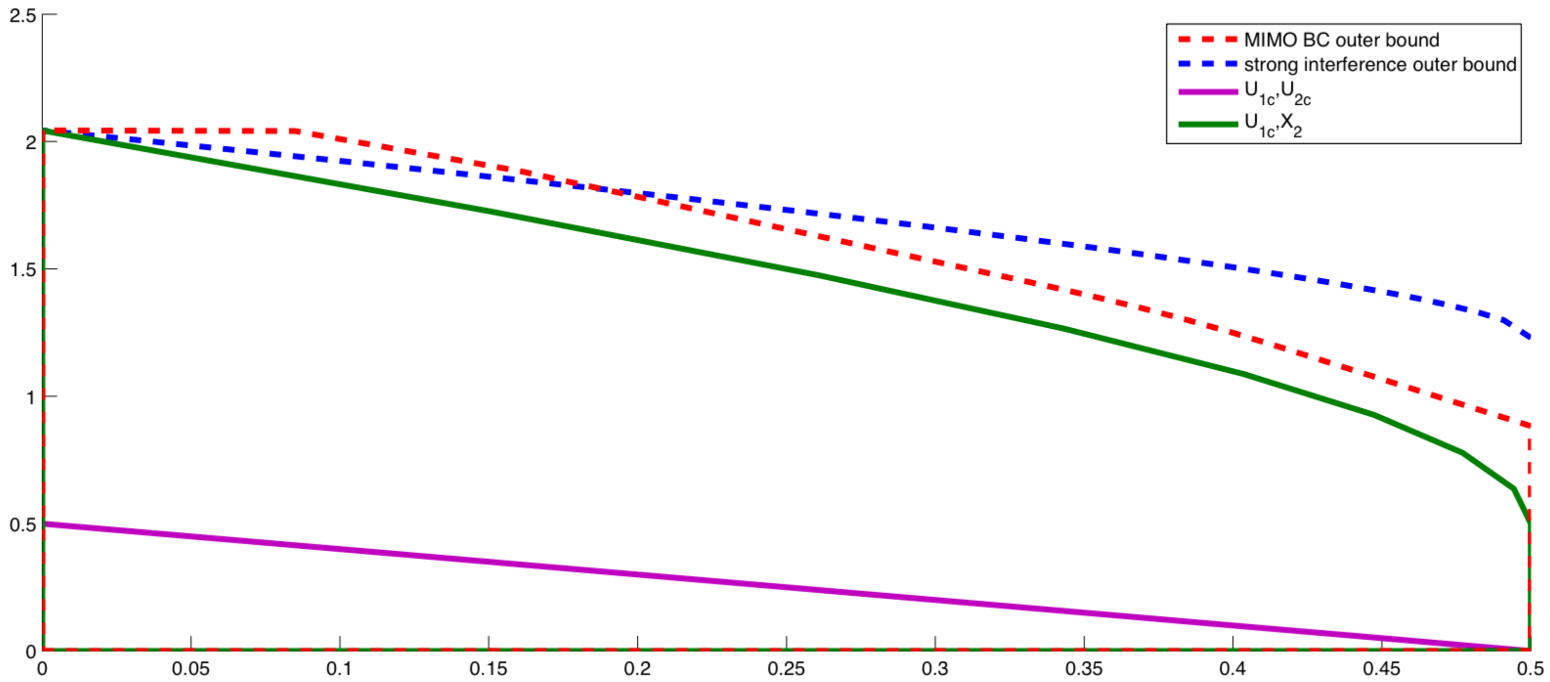
Achievability:

$$
\begin{aligned}
Y_1 &\sim U_{1pb} \\
&= X_1 + aX_2 + Z_1' \\
&\approx X_1 + aX_2 \\
&\approx X_1 + \frac{P_1}{P_1 + 1} aX_2
\end{aligned}
$$

Gap



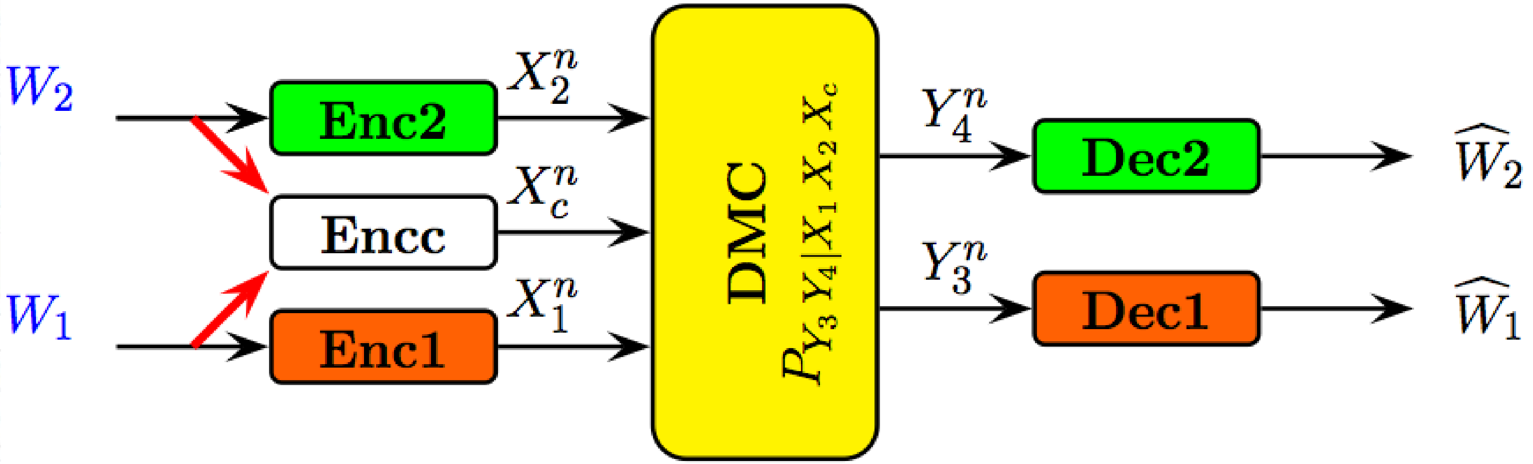
Z-IFC b=3 a=0



2.5
2
1.5
1
0.5
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
MIMO BC outer bound
strong interference outer bound
U_{1c},U_{2c}
U_{1c},X_2

Part II: IFC-CR

- Outer bounds
- Achievability for (certain) high SNR deterministic channels



State-of-the-art: IFC-CR

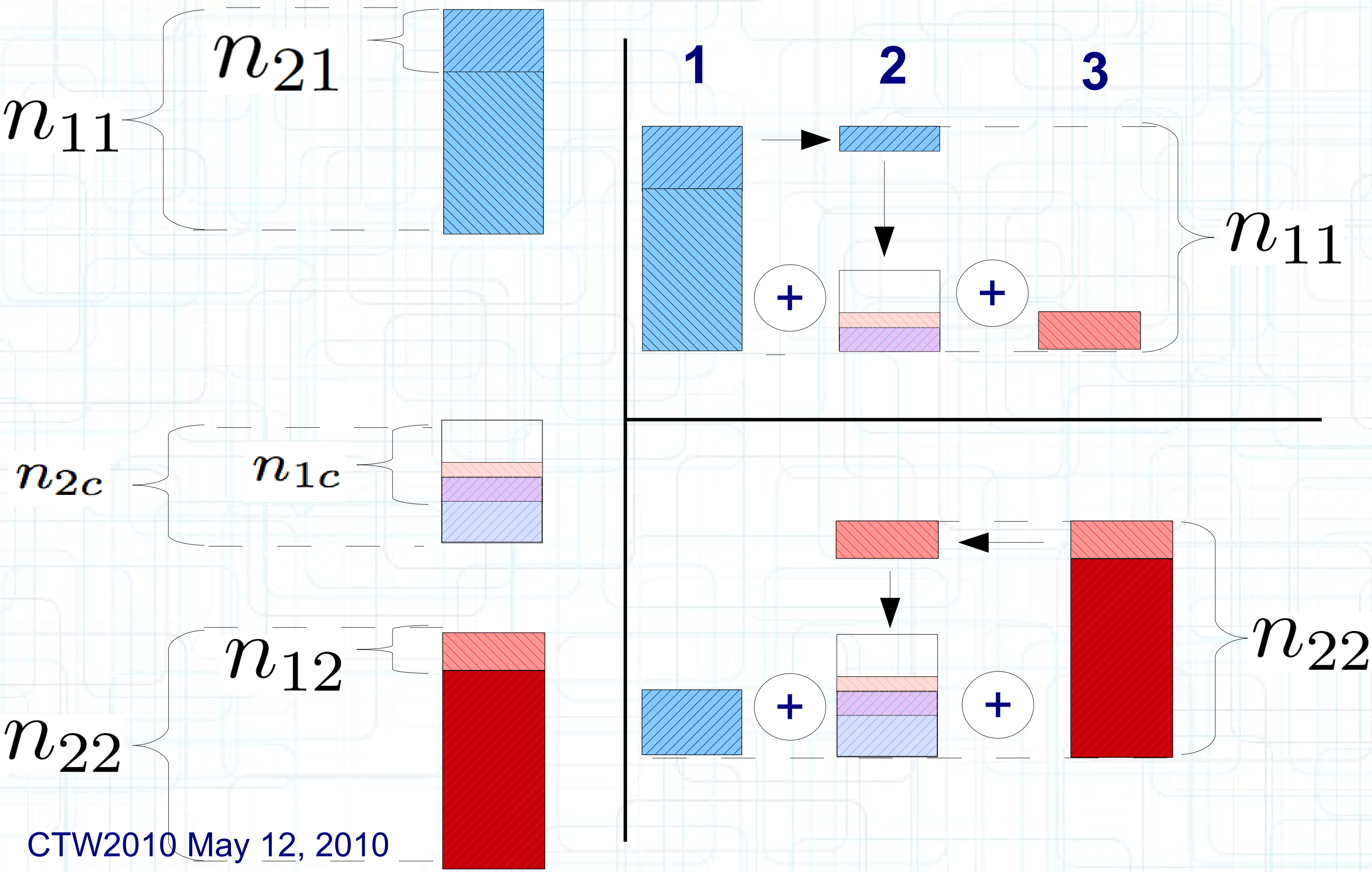
- 2007: **<u>Sahin</u>** *et al*
  - Model and inner bound.
- 2008: **<u>Sridharan</u>** *et al*
  - Inner and outer bound for Gaussian sumrate.
- 2009: **<u>Jiang</u>** *et al*
  - Inner bound.
- 2010: **<u>Rini</u>** *et al*
  - Outer bound, tight for certain high SNR deterministic channels.

Our contributions

- Outer bound for general channels (a la Sato's CSM)
- Tightened outer bound for a class of semi-deterministic channels
- Achievability for high SNR channels for all parameters we tried (not finished yet)

High SNR channel

- Interesting achievable schemes: $n_{11} > n_{1c} > n_{12}$ and $n_{22} > n_{2c} > n_{21}$ (R1<n11, R2<n22)



On going work

- High SNR channel: prove capacity for all parameter regimes
- (Semi) deterministic channel: capacity for certain classes
- Gaussian channel: compare our outer bound with existing ones (cognitive MIMO)
- Gaussian channel: finite gap

Conclusions

- C-IFC:
  - New inner bound, New Sato-type outer bound
  - Gap less 1.07 bits for Gaussian
  - Capacity for certain classes of channels
- IFC-CR:
  - New Sato-type outer bound
  - Capacity for high SNR channel for all parameter range we tried
  - Working on Gaussian channel
- IFC-GF:
  - New Sato-type outer bound
  - Working on reducing gap for Gaussian channel
  - Working on multiplicative gap

Thank you!!!