Robust Interference Alignment

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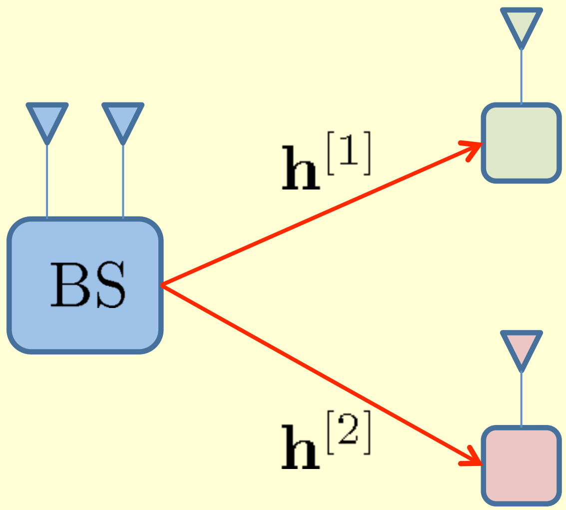
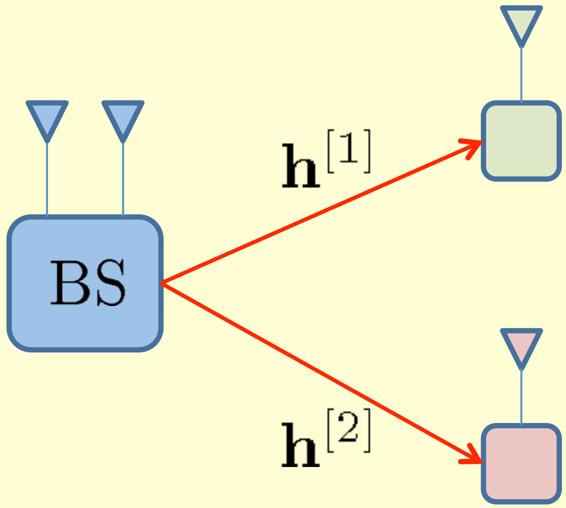
MISO Broadcast Channel



- Capacity known with perfect CSIT, CSIR [Weingarten, Steinberg, Shamai 03]
- Capacity unknown without full CSIT even for SISO setting.
- Even capacity-prelog (DoF) unknown for the MISO setting.

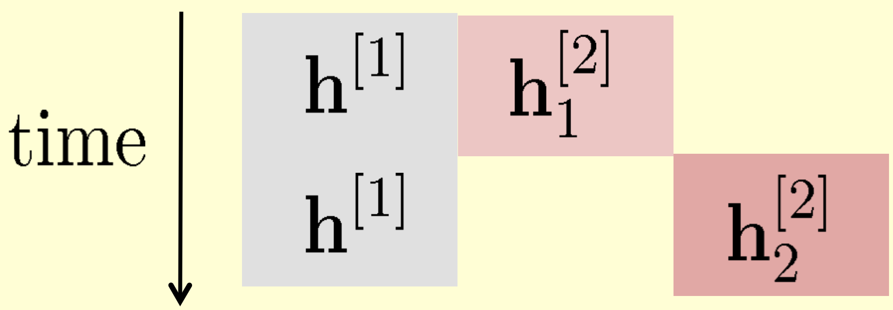
MISO BC DoF with Full CSIT

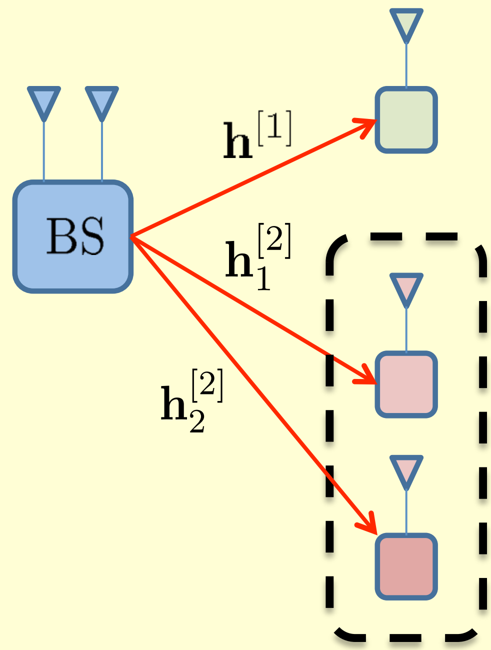


Can create a non-interfering channel to each user per time slot.

2 Users, 2 antennas at BS, DoF = 2

DoF of Compound MISO BC





$$\text{DoF} = 1 + \frac{1}{2} \qquad \text{(Optimal!)}$$

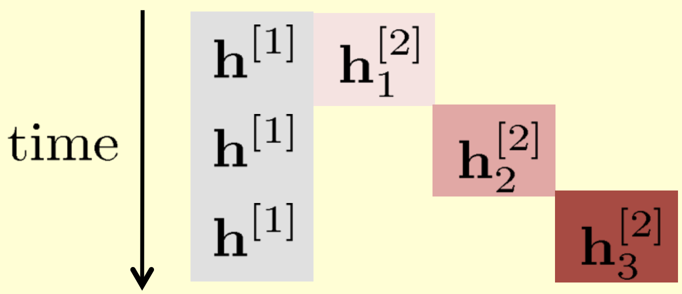
[Weingarten, Shamai, Kramer, ITA 07]

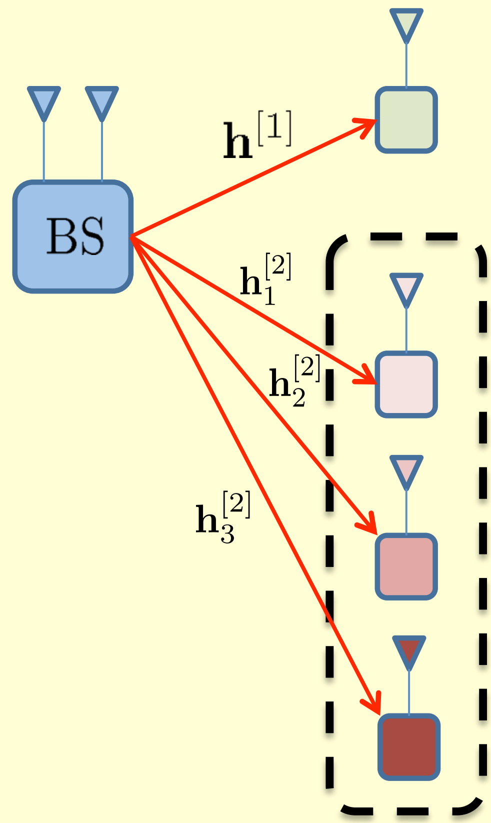
DoF of Compound MISO BC

[Weingarten, Shamai, Kramer, ITA 07]

Conjecture:

$\mathrm{DoF}=1+\frac{1}{3}$



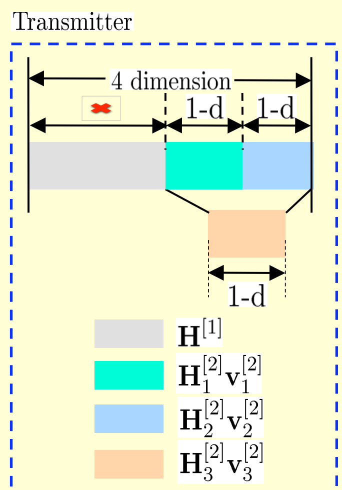


Best Outer Bound

$$\mathrm{DoF} \leq 1 + \frac{1}{2}$$

[Gou, Jafar, Wang, 09]

$\mathrm{DoF}=1+\frac{1}{2}$



$$\mathbf{H}_1^{[2]}\mathbf{v}_1^{[2]} + \mathbf{H}_2^{[2]}\mathbf{v}_2^{[2]} = \mathbf{H}_3^{[2]}\mathbf{v}_3^{[2]}$$

Interference Alignment
at the Transmitter

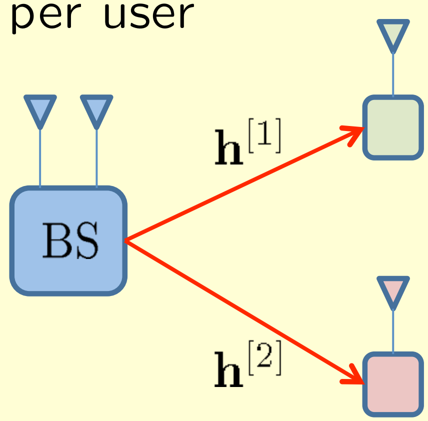
- Two symbol extension (constant channel).
- Linear beamforming.
- Asymmetric complex signaling.

DoF of Compound MISO BC

M antennas at BS, K Users, J generic states per user

[Gou, Jafar, Wang 09]
[Maddah-Ali 09]

$$\mathrm{DoF} = \frac{MK}{M+K-1}$$



- Does not depend on J provided $J \geq M$.
- Same as the X channel with M Tx, K Rx.
- No cooperation needed between Tx antennas.

$$\mathbf{h}^{[1]} \in \{\mathbf{h}_1^{[1]}, \mathbf{h}_2^{[1]}, \ldots, \mathbf{h}_{J_1}^{[1]}\}$$

$$\mathbf{h}^{[2]} \in \{\mathbf{h}_1^{[2]}, \mathbf{h}_2^{[2]}, \ldots, \mathbf{h}_{J_2}^{[2]}\}$$

Compound MISO BC reduces to Compound X Channel

Compound X Channel does not lose DoF. **Why?**

What is the main challenge of the compound setting?

How to Simultaneously Satisfy an Arbitrarily Large Number of Interference Alignment Conditions

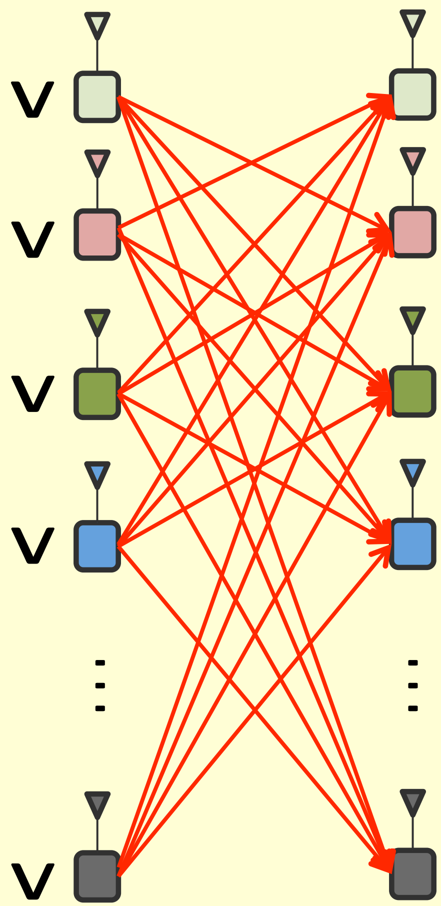
Have we seen this problem before ?

Yes, similar challenge is overcome for the K user interference channel [Cadambe, Jafar, IT Trans. 08]

Interference Alignment Scheme of [CJ08]

Every transmitter uses the SAME set of m beamforming vectors.

$$\mathbf{V} = [V_1, V_2, \cdots, V_m]$$



What is the interference space at Receiver 1 ?

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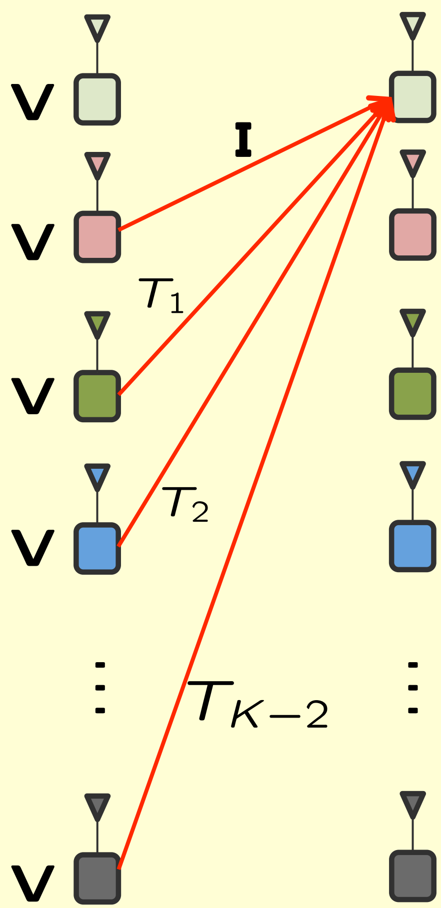

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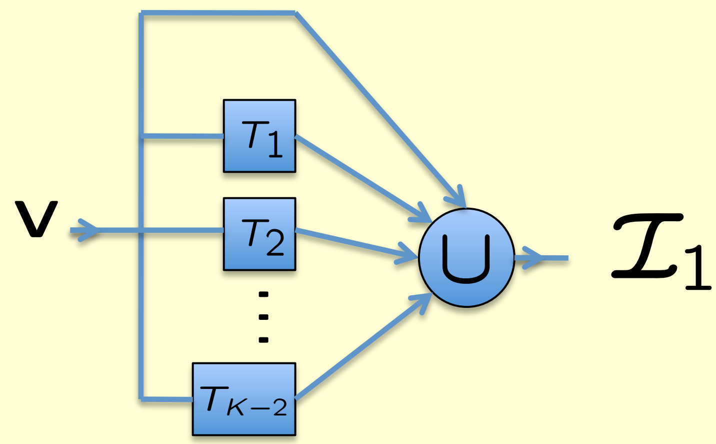

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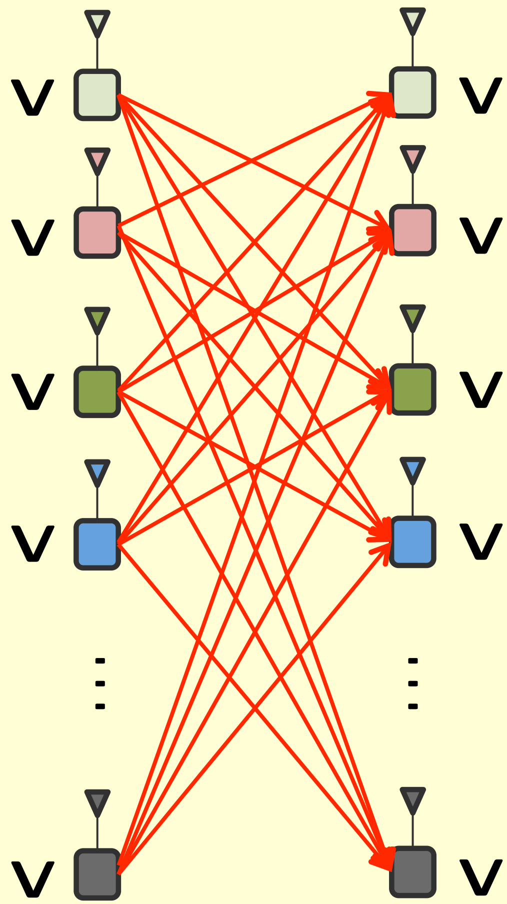
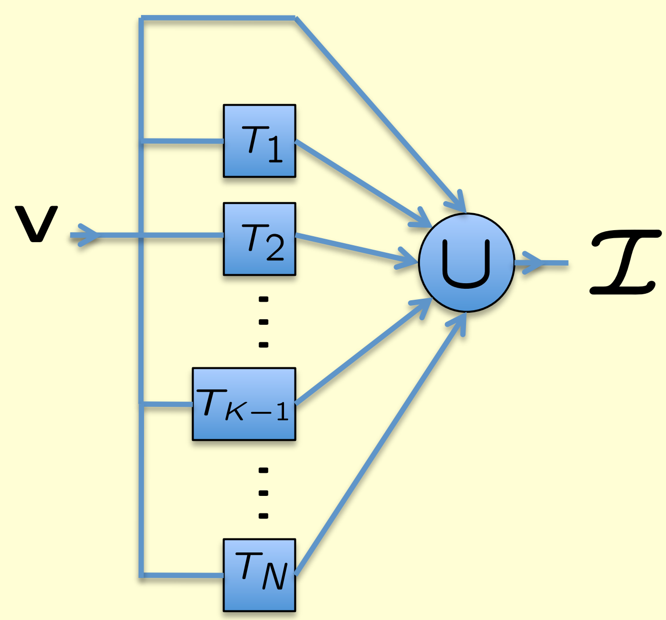


All the interference at all the receivers: $\mathcal{I}$



Goal: Make $\mathbf{V} \equiv \mathcal{I}$

Interference Alignment Scheme of [CJ08]

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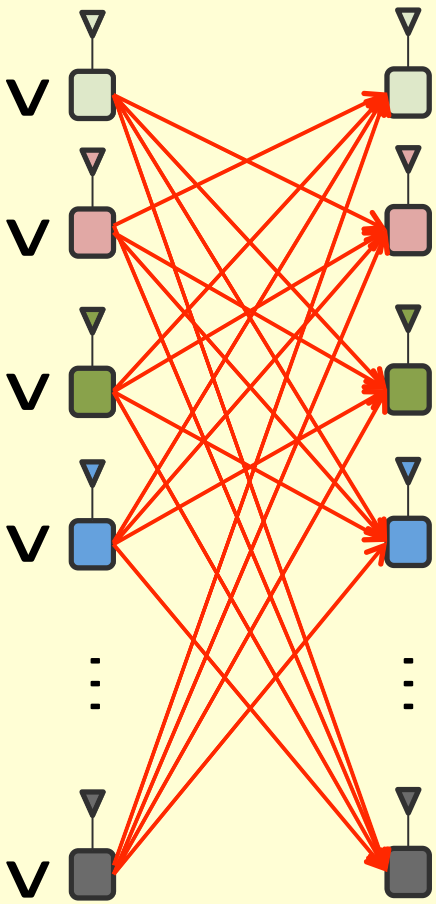
$$\mathbf{V} = [V_1, V_2, \cdots, V_m]$$



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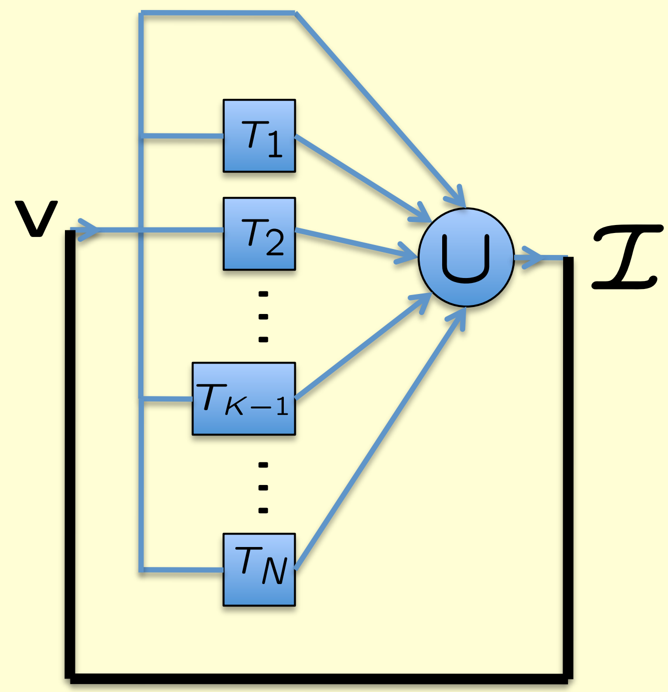
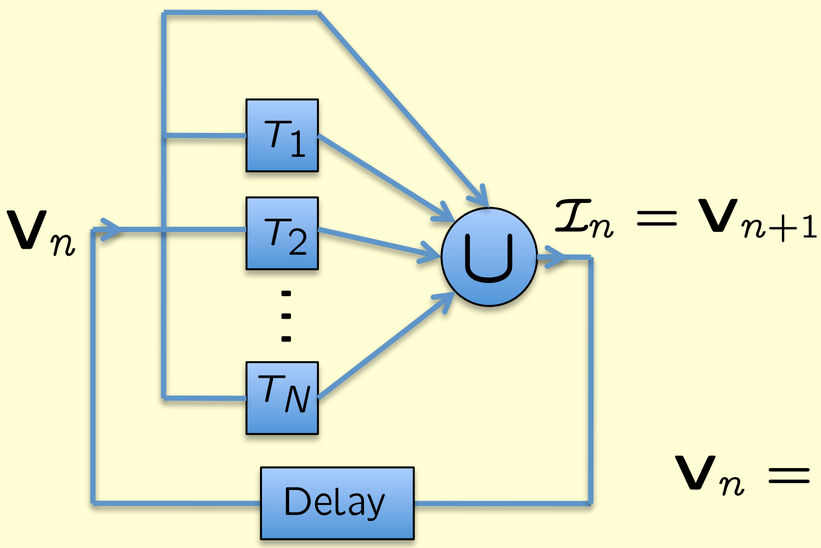


Goal: Make $\mathbf{V} \equiv \mathcal{I}$

Goal: Simultaneously satisfy "N" Alignment Constraints:
$\text{span}(\mathbf{V}) \equiv \text{span}(T_1\mathbf{V}) \equiv \text{span}(T_2\mathbf{V}) \equiv \cdots \equiv \text{span}(T_N\mathbf{V})$



Initialize: $\mathbf{V}_0 = \mathbf{1}$

$$\mathbf{V}_1 = \{\mathbf{1}, T_1\mathbf{1}, \cdots, T_N\mathbf{1}\}$$

$$\mathbf{V}_2 = \{\mathbf{1}, \cdots, T_i\mathbf{1}, \cdots, T_iT_j\mathbf{1}, \cdots, T_i^2\mathbf{1}\}$$

$$\mathbf{V}_n = \{T_1^{\alpha_1}T_2^{\alpha_2}\cdots T_N^{\alpha_N}\mathbf{1}, \alpha_1 + \cdots + \alpha_N \leq n\}$$

$$\mathbf{V}_{n+1} = \{T_1^{\alpha_1}T_2^{\alpha_2}\cdots T_N^{\alpha_N}\mathbf{1}, \alpha_1 + \cdots + \alpha_N \leq n+1\}$$

$$|\mathbf{V}_n| = \binom{n+N}{n}$$

$$|\mathcal{I}| = \binom{n+N+1}{n+1}$$

$$\frac{|\mathbf{V}|}{|\mathcal{I}|} = \frac{n+1}{n+N+1} \to 1 \text{ as } n \to \infty$$

Compound setting originally intended to capture channel uncertainty,

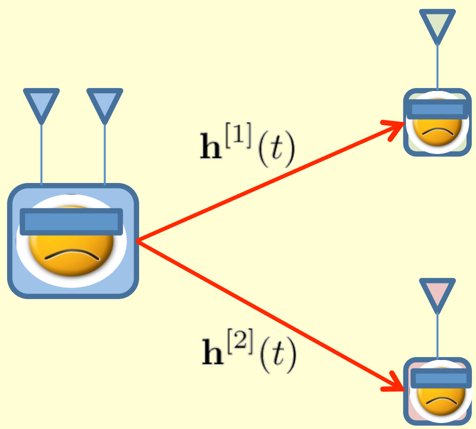
to show loss of DoF

Interference Alignment scheme of [CJ08]

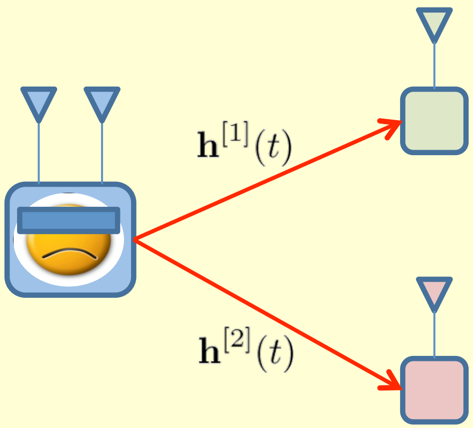
- **Strength** - Unlimited alignment potential (DoF do not collapse).
- **Weakness** - Needs perfect knowledge of all states.

ROBUST interference alignment ?

What if instantaneous channel coefficient values are
completely unknown
to transmitter (and receivers)?



Blind Interference Alignment

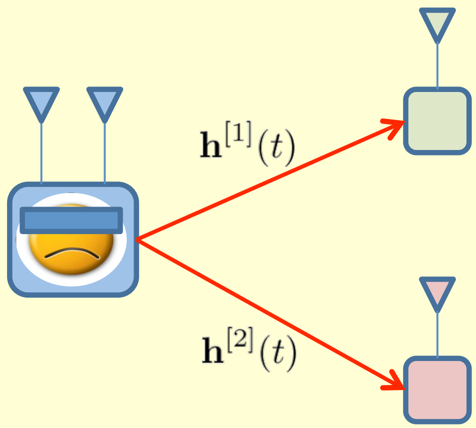


Users are **statistically indistinguishable**
DoF $= 1$
[Caire, Shamai '00, Jafar '05]

Users are **not statistically indistinguishable**

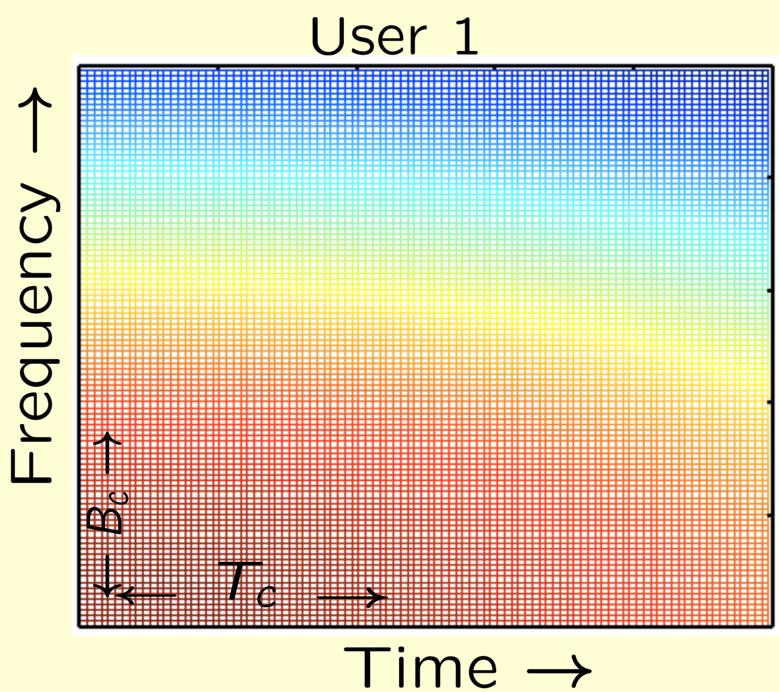
[Lapidoth, Shamai, Wigger, Allerton 05]
DoF $\leq \frac{4}{3}$
Conjecture: DoF $= 1$.
(No multiplexing of signals is possible)

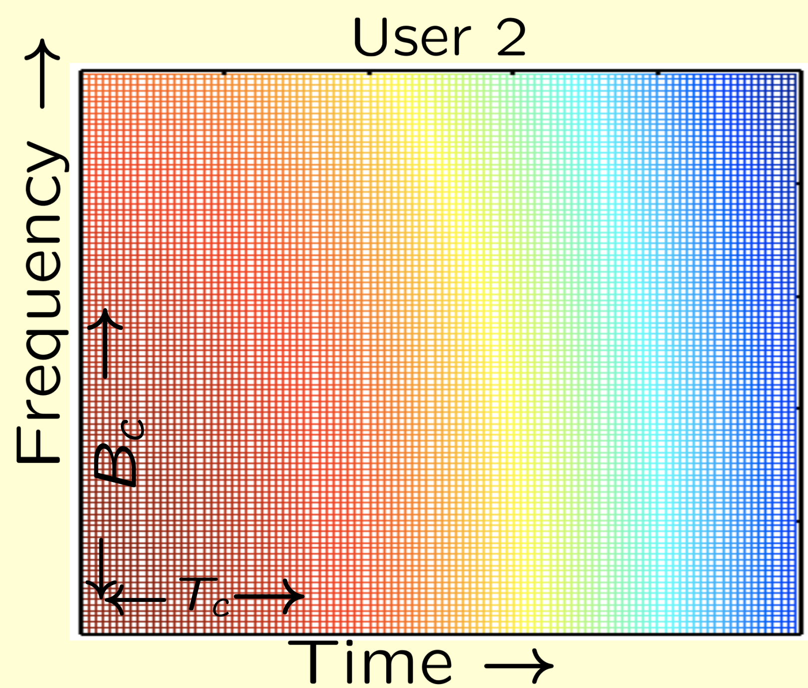
What does the transmitter know?

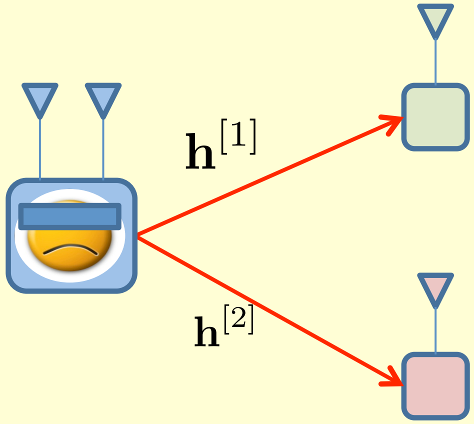


Knows the channel statistics.
Knows the fading **autocorrelation** function.

Goal: Achieve $\frac{4}{3}$ DoF

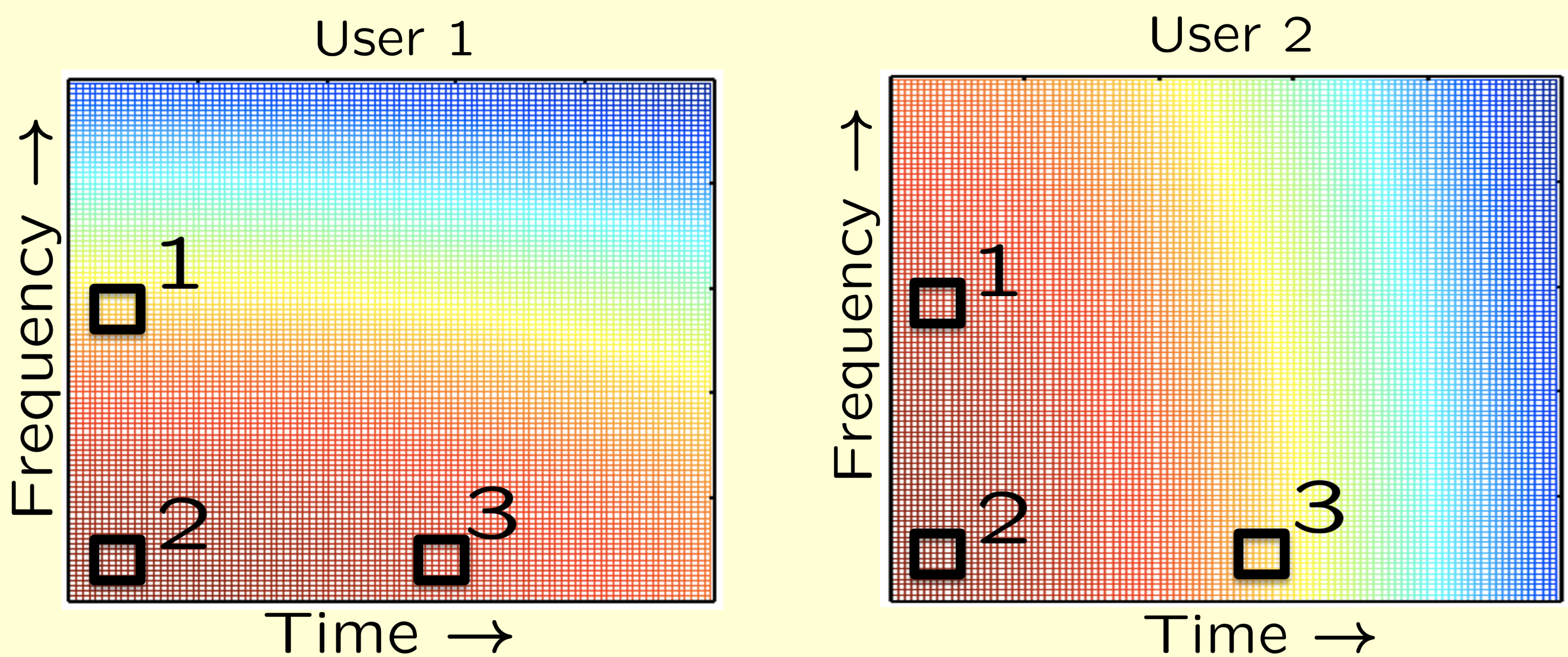






3 Dimensional Signaling

	1	2	3
User 1	$\mathbf{h}^{[1]}(1)$	$\mathbf{h}^{[1]}(2)$	$\mathbf{h}^{[1]}(2)$
User 2	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(2)$



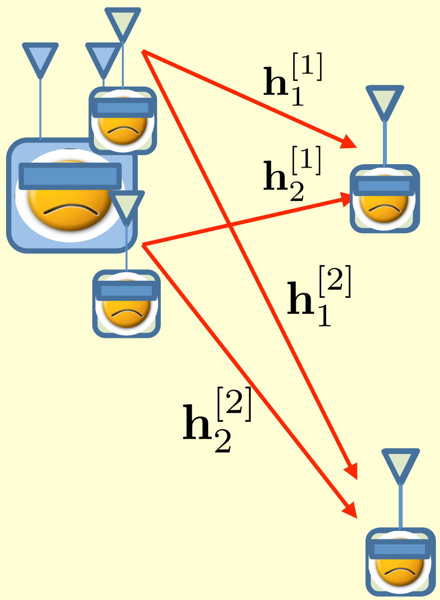
3 Dimensional Signaling

Goal
Send 4 interference-free signals over three dimensions

	1	2	3
User 1	$\mathbf{h}^{[1]}(1)$	$\mathbf{h}^{[1]}(2)$	$\mathbf{h}^{[1]}(2)$
User 2	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(2)$

Transmit antennas do not need to share data.

Receivers do $\bar{\text{not}}$ need CSIR to remove interference.



3 Dimensional Signaling

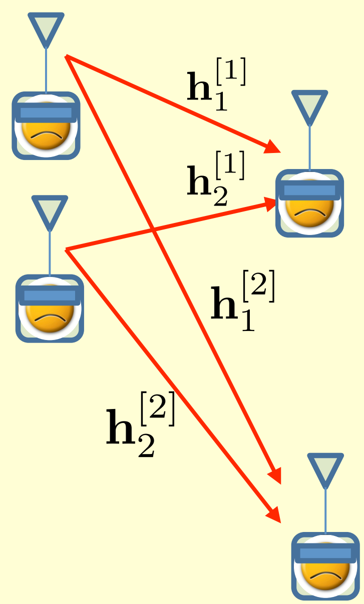
Goal
Send 4 interference-free signals over three dimensions

	1	2	3
User 1	$\mathbf{h}^{[1]}(1)$	$\mathbf{h}^{[1]}(2)$	$\mathbf{h}^{[1]}(2)$
User 2	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(2)$

Transmit antennas do not need to share data.

Receivers do not need CSIR to remove interference.

$t=3$	$t=2$	$t=1$
$x_1^{[2]}$	$x_1^{[1]} + x_1^{[2]}$	$x_1^{[1]}$
$x_2^{[2]}$	$x_2^{[1]} + x_2^{[2]}$	$x_2^{[1]}$



$t=1$	$t=2$	$t=3$
$h_1^{[1]}(1)x_1^{[1]} + h_2^{[1]}(1)x_1^{[1]}$	$h_1^{[1]}(2)x_1^{[1]} + h_1^{[1]}(2)x_1^{[2]} + h_2^{[1]}(2)x_1^{[1]} + h_2^{[1]}(2)x_2^{[2]}$	$h_1^{[1]}(2)x_1^{[2]} + h_2^{[1]}(2)x_2^{[2]}$
$h_1^{[2]}(1)x_1^{[1]} + h_2^{[2]}(1)x_2^{[1]}$	$h_1^{[2]}(1)x_1^{[1]} + h_1^{[2]}(1)x_1^{[2]} + h_2^{[2]}(1)x_1^{[1]} + h_2^{[2]}(1)x_2^{[2]}$	$h_1^{[2]}(2)x_1^{[2]} + h_2^{[2]}(2)x_2^{[2]}$

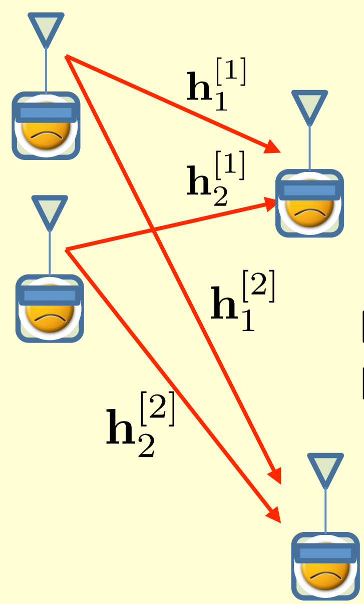
3 Dimensional Signaling

Goal
Send 4 interference-free signals over three dimensions

	1	2	3
User 1	$\mathbf{h}^{[1]}(1)$	$\mathbf{h}^{[1]}(2)$	$\mathbf{h}^{[1]}(2)$
User 2	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(2)$

Transmit antennas do not need to share data.

$t=3$	$t=2$	$t=1$
$x_1^{[2]}$	$x_1^{[1]}+x_1^{[2]}$	$x_1^{[1]}$
$x_2^{[2]}$	$x_2^{[1]}+x_2^{[2]}$	$x_2^{[1]}$



$$\begin{bmatrix} y^{[1]}(1) \\ y^{[1]}(2)' \end{bmatrix} = \begin{bmatrix} h_1^{[1]}(1) & h_2^{[1]}(1) \\ h_1^{[1]}(2) & h_2^{[1]}(2) \end{bmatrix} \begin{bmatrix} x_1^{[1]} \\ x_2^{[1]} \end{bmatrix} + \begin{bmatrix} n^{[1]}(1) \\ n^{[1]}(2)' \end{bmatrix}$$

Each user sees 2×2 MIMO channel
No interference

$$\begin{bmatrix} y^{[2]}(2)' \\ y^{[2]}(3) \end{bmatrix} = \begin{bmatrix} h_1^{[2]}(1) & h_2^{[2]}(1) \\ h_1^{[2]}(2) & h_2^{[2]}(2) \end{bmatrix} \begin{bmatrix} x_1^{[2]} \\ x_2^{[2]} \end{bmatrix} + \begin{bmatrix} n^{[1]}(2)' \\ n^{[1]}(3) \end{bmatrix}$$

Intuition for Blind Interference Alignment

M transmit antennas.
Each sends an independent symbol.
Repeat **same** symbol M times.
One receive antenna.

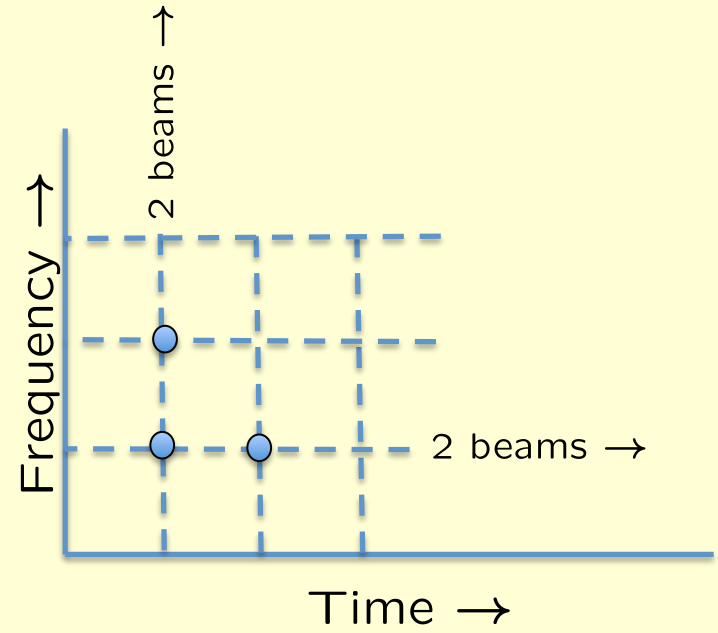
Channel changes every time
Receive M independent linear equations.
Symbols can be resolved ($M \times M$ MIMO)

Channel stays constant
Receive same equation M times.
Symbols cannot be resolved.
(**Interference Alignment**)

Idea - Repeat symbols where desired users' channel changes undesired users' channels remain the same.

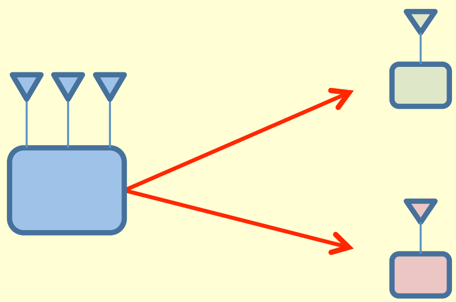
Idea - Repeat symbols where desired users' channel changes undesired users' channels remain the same.

	1	2	3
User 1	$\mathbf{h}^{[1]}(1)$	$\mathbf{h}^{[1]}(2)$	$\mathbf{h}^{[1]}(2)$
User 2	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(1)$	$\mathbf{h}^{[2]}(2)$

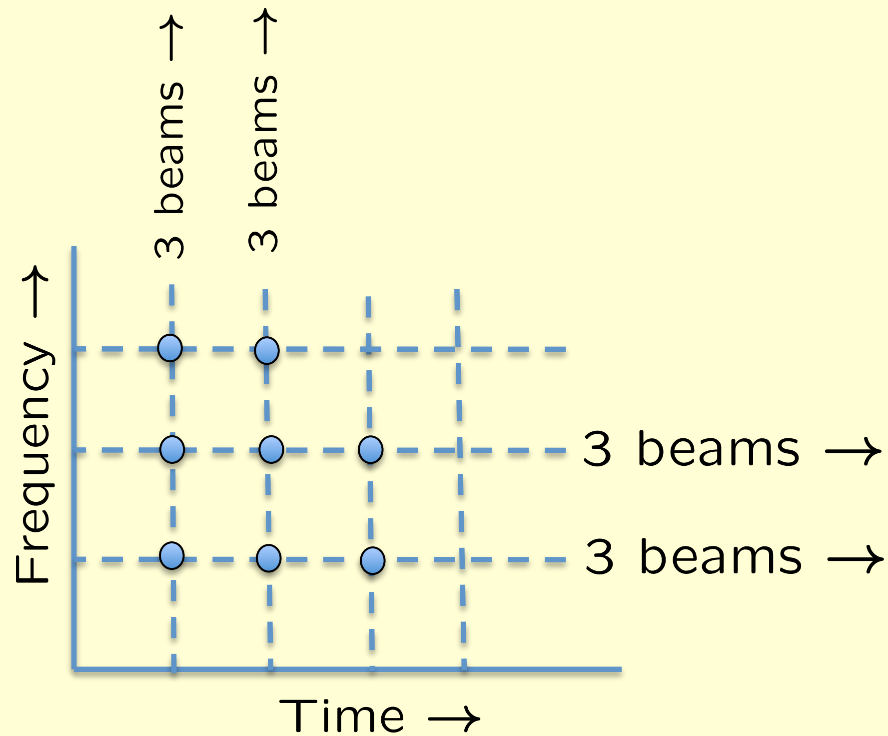


$t=1$	$t=2$	$t=3$
$x_1^{[1]}$	$x_1^{[1]}+x_1^{[2]}$	$x_1^{[2]}$
$x_2^{[1]}$	$x_2^{[1]}+x_2^{[2]}$	$x_2^{[2]}$

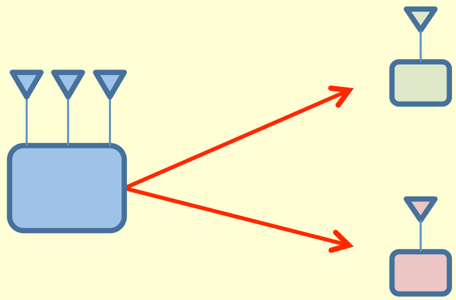
2 user 3×1 MISO BC



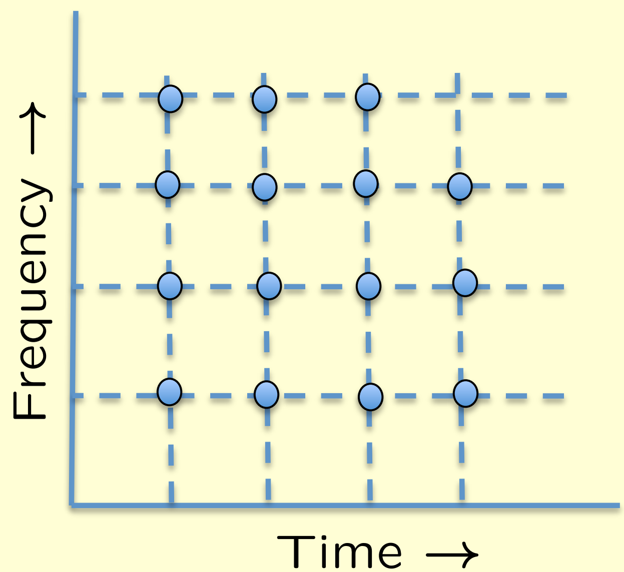
$$\text{DoF} = \frac{2 \times 3}{2+3-1} = \frac{12}{8}$$



2 user $M \times 1$ MISO BC



$$\text{DoF} = \frac{2 \times M \times (M-1)}{M^2 - 1} = \frac{2M}{M+1}$$



Conclusion

We asked for a lot

- Transmitter does **not** have CSIT **can still align interference**
- Receiver does **not** have CSIR **can still cancel interference**
- Transmit antennas can be **distributed**

... and it still works!

Robust Interference Alignment is Possible

... and the dream lives on