A Game-Theoretic Perspective of the Interference Channel: Impact of Coordination and Bargaining

X. Liu, E. Erkip

Polytechnic Institute of New York University

May 2010
Wireless systems often limited by *interference*.
  Modeled by the *interference channel (IC)*

Usual information theoretical approach:
  *Full cooperation* among users for codebook and rate selection, e.g. Han-Kobayashi (H-K) scheme

In practice, users may be *selfish* and only interested in maximizing their own utilities (rates).

When there is no coordination, interference is often treated as noise, *suboptimal* in most cases.

What if users are selfish but willing to coordinate?
For users with conflicting interests, achieving efficiency and fairness can be studied using game theory.

Two common game theoretical approaches:
- Noncooperative game theory
- Cooperative game theory
  - Nash bargaining solution (NBS)
Our Approach

- Assume each user is selfish but willing to coordinate only when an incentive exists.
- Formulate interaction between users as a bargaining problem.
- Allow users to adopt a simple H-K type scheme with an optimal (or close to optimal) fixed power split.

Two-phase coordination

- Phase 1: Users negotiate and decide to use the H-K scheme only if both have incentives.
- Phase 2: The operating point on the H-K region is selected using NBS from cooperative game theory.
Gaussian interference games using noncooperative game theory, and assuming interference is treated as noise [Etkin et al 07, Larsson et al 08].

Noncooperative information theoretical games assuming each user can select any encoding and decoding strategy [Berry and Tse 08, 09].

Noncooperative rate game over a Gaussian MAC [Gajic et al 08].

NBS for interfering links in multi-cell OFDMA [Han et al 05].

NBS for an orthogonal scheme (TDM/FDM) over fading IC [Leshem et al 08].
Two-user Gaussian IC:

\[ Y_1 = X_1 + \sqrt{a}X_2 + Z_1 \]
\[ Y_2 = \sqrt{b}X_1 + X_2 + Z_2 \]
Assumptions:

- Users employ Gaussian codebooks with equal length codewords.
- A simplified H-K type scheme with a fixed power split and no time-sharing.

\( \alpha \in [0, 1] \) and \( \beta \in [0, 1] \): Private message power ratios of user 1 and user 2 respectively.

\( \mathcal{F} \): Achievable rate pairs \((R_1, R_2) \in \mathbb{R}^2_+\)
Achievable Rate Region $\mathcal{F}$

\[
R_1 \leq \phi_1 = C \left( \frac{P_1}{1 + a \beta P_2} \right)
\]

\[
R_2 \leq \phi_2 = C \left( \frac{P_2}{1 + b \alpha P_1} \right)
\]

\[
R_1 + R_2 \leq \phi_3 = \min \{ \phi_{31}, \phi_{32}, \phi_{33} \}
\]

with

\[
\phi_{31} = C \left( \frac{P_1 + a(1 - \beta)P_2}{1 + a \beta P_2} \right) + C \left( \frac{\beta P_2}{1 + b \alpha P_1} \right)
\]

\[
\phi_{32} = C \left( \frac{\alpha P_1}{1 + a \beta P_2} \right) + C \left( \frac{P_2 + b(1 - \alpha)P_1}{1 + b \alpha P_1} \right)
\]

\[
\phi_{33} = C \left( \frac{\alpha P_1 + a(1 - \beta)P_2}{1 + a \beta P_2} \right) + C \left( \frac{\beta P_2 + b(1 - \alpha)P_1}{1 + b \alpha P_1} \right)
\]

\[
2R_1 + R_2 \leq \phi_4 = C \left( \frac{P_1 + a(1 - \beta)P_2}{1 + a \beta P_2} \right) + C \left( \frac{\alpha P_1}{1 + a \beta P_2} \right) + C \left( \frac{\beta P_2 + b(1 - \alpha)P_1}{1 + b \alpha P_1} \right)
\]

\[
R_1 + 2R_2 \leq \phi_5 = C \left( \frac{P_2 + b(1 - \alpha)P_1}{1 + b \alpha P_1} \right) + C \left( \frac{\beta P_2}{1 + b \alpha P_1} \right) + C \left( \frac{\alpha P_1 + a(1 - \beta)P_2}{1 + a \beta P_2} \right)
\]

where $C(x) = \frac{1}{2} \log_2 (1 + x)$. 
Denote the H-K scheme that achieves the above rate region \( F \) by HK\((\alpha, \beta)\).

\( F \) can be represented in a matrix form as
\[
F = \{ R | R \geq 0, R \leq R^1, \text{ and } AR \leq B \},
\]
where \( R = (R_1 \ R_2)^t \),
\[
R^1 = (\phi_1 \ \phi_2)^t,
\]
\[
B = (\phi_3 \ \phi_4 \ \phi_5)^t,
\]
and
\[
A = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}^t
\]
Bargaining Problem for Rates

- **Feasible set:**
  - Set of all possible agreements that users can jointly achieve.
  - The rate region \( F \) achieved by HK(\( \alpha, \beta \)).

- **Disagreement point:**
  - Rate allocation that results when users fail to agree.
  - Each user treats the other’s signal as noise
    \[
    R^0 = \left( C\left( \frac{P_1}{1+aP_2} \right) \; C\left( \frac{P_2}{1+bP_1} \right) \right)^t.
    \]
  - Bargaining problem represented by \((F, R^0)\).

**Definition:** \((F, R^0)\) is essential iff \( F \cap \{R \mid R > R^0 \} \) is nonempty.
\( R^* = \Phi(\mathcal{F}, R^0) \) is a NBS for \((\mathcal{F}, R^0)\), if the following are satisfied:

1. **Individual Rationality:** \( \Phi_i(\mathcal{F}, R^0) \geq R^0_i, \forall i \)
2. **Feasibility:** \( \Phi(\mathcal{F}, R^0) \in \mathcal{F} \)
3. **Pareto Optimality:** \( \Phi(\mathcal{F}, R^0) \) is Pareto optimal.
4. **Independence of Irrelevant Alternatives:** For any closed convex set \( \mathcal{G} \), if \( \mathcal{G} \subseteq \mathcal{F} \) and \( \Phi(\mathcal{F}, R^0) \in \mathcal{G} \), then \( \Phi(\mathcal{G}, R^0) = \Phi(\mathcal{F}, R^0) \).
5. **Scale Invariance:** For any numbers \( \lambda_1, \lambda_2, \gamma_1 \) and \( \gamma_2 \), such that \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \), if \( \mathcal{G} = \{(\lambda_1 R_1 + \gamma_1, \lambda_2 R_2 + \gamma_2) | (R_1, R_2) \in \mathcal{F}\} \) and \( g^0 = (\lambda_1 R^0_1 + \gamma_1, \lambda_2 R^0_2 + \gamma_2) \), then \( \Phi(\mathcal{G}, g^0) = (\lambda_1 \Phi_1(\mathcal{F}, R^0) + \gamma_1, \lambda_2 \Phi_2(\mathcal{F}, R^0) + \gamma_2) \).
6. **Symmetry:** If \( R^0_1 = R^0_2 \), and \( \{(R_2, R_1) | (R_1, R_2) \in \mathcal{F}\} = \mathcal{F} \), then \( \Phi_1(\mathcal{F}, R^0) = \Phi_2(\mathcal{F}, R^0) \).
The unique solution $\Phi(\mathcal{F}, R^0)$ satisfying all six axioms above is given by,

$$
\Phi(\mathcal{F}, R^0) = \arg \max_{R \in \mathcal{F}, R \geq R^0} \prod_{i=1}^{2} (R_i - R_i^0)
$$
The capacity region $C$ is given by $(R_1, R_2)$ such that

$$R_i \leq C(P_i), \ i \in \{1, 2\}$$

$$R_1 + R_2 \leq \phi_0 = C(P_1 + P_2)$$

If users fully cooperate, any rate pair is achievable.

Treating the other user’s signal as noise leads to rate

$$R_i^0 = C\left(\frac{P_i}{1+P_{3-i}}\right) \text{ for user } i.$$  

Use as the disagreement point.

Nash bargaining problem: $(C, R^0)$
Proposition

There exists a unique NBS for the bargaining problem \((C, R^0)\),
given by \(R^* = (R_1^0 + \frac{1}{\mu_1}, R_2^0 + \frac{1}{\mu_1})\) where \(\mu_1 = \frac{2}{\phi_0 - R_1^0 - R_2^0}\).

Proof: Formulate the Nash optimization problem and invoke the KKT conditions.
SNR_1 = 15dB, SNR_2 = 20dB
Two-Phase Mechanism for Gaussian IC

- **Phase 1:**
  - Negotiate for a simple H-K scheme that has the potential to improve individual rates for both.
  - Negotiation breakdown if one user does not have an incentive to cooperate.

- **Phase 2:**
  - Bargain for a fair rate pair over the achievable rate region of the H-K scheme agreed on in Phase 1.
Phase 1 Incentive Conditions: Strong Interference

- $a \geq 1$ and $b \geq 1$:
  - Choose optimal $\alpha = \beta = 0$.
  - Bargaining problem $(\mathcal{F}, \mathbf{R}^0)$ is essential, both users always have incentives to cooperate.
Phase 1 Incentive Conditions: Weak Interference

- \( a < 1 \) and \( b < 1 \):
  - Use power splits \( \alpha = \min\left(\frac{1}{bP_1}, 1\right) \) and \( \beta = \min\left(\frac{1}{aP_2}, 1\right) \).
  - At most 1-bit away from the capacity [Etkin, Tse, Wang 08]
  - For \( bP_1 \leq 1 \), \( HK(1, \beta) \) doesn't improve user 2's rate
    - User 2 does not have an incentive to cooperate.
  - Both users can improve upon \( R^0 \) and agree to cooperate only when
    - \( aP_2 > 1 \), \( bP_1 > 1 \) and \( \mathcal{F} \cap \{ R > R^0 \} \) is nonempty for \( HK\left(\frac{1}{bP_1}\right) \frac{1}{aP_2} \),
Phase 1 Incentive Conditions: Mixed Interference

- $a < 1$ and $b \geq 1$:
  - Use near-optimal power splits $\alpha = 0$ and $\beta = \min(1/(aP_2), 1)$.
  - Similar to the weak case, both users agree to cooperate if $aP_2 > 1$ and $\mathcal{F} \cap \{R > R^0\}$ is nonempty for $\text{HK}(0, 1/(aP_2))$. 
Phase 2: NBS over IC

**Proposition**

Assuming that $R^0 < R^1$ and $AR^0 < B$, there exists a unique NBS $R^*$ for the bargaining problem $(\mathcal{F}, R^0)$, and is characterized as follows:

$$R^*_i = \min \left\{ R^1_i; R^0_i + \frac{1}{\sum_{j=1}^{3} \mu_j A_{ji}} \right\}, \quad i \in \{1, 2\}$$

where $\mu_j \geq 0$, $j \in \{1, 2, 3\}$ is chosen to satisfy

$$(AR^* - B)_j \mu_j = 0, \quad AR^* \leq B$$
Strong Interference

\[ a = 3, \ b = 5, \ SNR_1 = 20\text{dB}, \ SNR_2 = 20\text{dB} \]
Mixed Interference

\[ a = 0.1, \ b = 3, \ \text{SNR}_1 = 20\text{dB}, \ \text{SNR}_2 = 20\text{dB} \]
Weak Interference

\[ a = 0.2, \ b = 0.5, \ SNR_1 = 20\text{dB}, \ SNR_2 = 20\text{dB} \]
NBS ($R^*$) and disagreement point ($R^0$), $\text{SNR}_1 = \text{SNR}_2 = 20\text{dB}$, $a = 1.5$. 

Rates versus Interference
Sum Rate versus Interference

SNR$_1$ = SNR$_2$ = 20dB, $a = 1.5$. 

\begin{align*}
\text{(H–K NBS)} \\
\text{(TDM NBS)} \\
\text{(H–K Sum Rate)}
\end{align*}
Some Limitations of the NBS Approach

- Most information concerning the bargaining environment and procedure is abstracted away.
- Each user’s cost of delay in bargaining is not taken into account.
- We also consider the strategic approach of *dynamic alternating-offer bargaining games (AOBG)*.
Features of the AOBG

- Model bargaining process as a noncooperative multi-stage game.
- Users alternate making offers in feasible set $\mathcal{F}$ until one is accepted.
- Cost of bargaining: An exogenous probability characterizing the risk of breakdown of bargaining due to some outside intervention.
User 1 and user 2 alternate making offers.
If user 2 rejects the offer made by user 1, there is a probability $p_1$ that the bargaining will end in the disagreement $R^0$.
Similarly, define $p_2$.
Bargaining continues until some offer is accepted or the game ends in disagreement.
When an offer is accepted, the users get the rates specified in the accepted offer.
Equilibrium of the AOBG

Proposition

For any regular two-user bargaining problem \((\mathcal{F}, R^0)\), let \((\tilde{R}, \tilde{\tilde{R}})\) be the unique pair of efficient agreements in \(\mathcal{F}\) which satisfy

\[
\tilde{R}_1 = (1 - p_2)(\tilde{R}_1 - R^0_1) + R^0_1
\]

\[
\tilde{R}_2 = (1 - p_1)(\tilde{R}_2 - R^0_2) + R^0_2
\]

In the equilibrium, user 1 always proposes \(\tilde{R}\) and accepts \(R\) with \(R_1 \geq \tilde{R}_1\); user 2 always proposes \(\tilde{R}\) and accepts \(R\) with \(R_2 \geq \tilde{R}_2\). Therefore, in equilibrium, the game will end in an agreement on \(\tilde{R}\) at round 1.
a = 0.2, b = 1.2, SNR_1 = 10dB and SNR_2 = 20dB
Rate versus $p_1$ in AOBG

$p_2 = 0.5$, $a = 0.2$, $b = 1.2$, SNR$_1 = 10$dB and SNR$_2 = 20$dB
Conclusion

- Coordination and bargaining can improve selfish users’ rate substantially compared with the uncoordinated case.
- NBS based on a simple H-K scheme not only provides a fair operating point but also maintains a good overall performance.
- AOBG models the bargaining process.
- Cost of bargaining: Risk of bargaining breakdown.
- Ongoing and future work:
  - Bargaining for degrees of freedom.
  - Other costs of bargaining: Reduction in utility.