A Game-Theoretic Perspective of the Interference Channel: Impact of Coordination and Bargaining

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Motivation

- Wireless systems often limited by *interference*.
  - Modeled by the *interference channel (IC)*
- Usual information theoretical approach:
  - *Full cooperation* among users for codebook and rate selection, e.g. Han-Kobayashi (H-K) scheme
- In practice, users may be *selfish* and only interested in maximizing their own utilities (rates).
- When there is no coordination, interference is often treated as noise, *suboptimal* in most cases.
- What if users are selfish but willing to coordinate?

Motivation for Game Theory

- For users with conflicting interests, achieving efficiency and fairness can be studied using game theory.
- Two common game theoretical approaches:
  - Noncooperative game theory
  - Cooperative game theory
    - Nash bargaining solution (NBS)

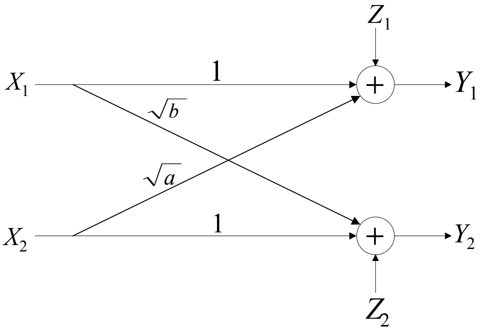
Our Approach

- Assume each user is selfish but willing to coordinate only when an incentive exists.
- Formulate interaction between users as a bargaining problem.
- Allow users to adopt a simple H-K type scheme with an optimal (or close to optimal) fixed power split.
- Two-phase coordination
  - Phase 1: Users negotiate and decide to use the H-K scheme only if both have incentives.
  - Phase 2: The operating point on the H-K region is selected using NBS from cooperative game theory.

Background: Interference and Game Theory

- Gaussian interference games using noncooperative game theory, and assuming interference is treated as noise [Etkin et al 07, Larsson et al 08].
- Noncooperative information theoretical games assuming each user can select any encoding and decoding strategy [Berry and Tse 08, 09].
- Noncooperative rate game over a Gaussian MAC [Gajic et al 08].
- NBS for interfering links in multi-cell OFDMA [Han et al 05].
- NBS for an orthogonal scheme (TDM/FDM) over fading IC [Leshem et al 08].

Channel Model



- Two-user Gaussian IC:

$$Y_1 = X_1 + \sqrt{a}X_2 + Z_1$$
$$Y_2 = \sqrt{b}X_1 + X_2 + Z_2$$

Achievable Rate Region

- Assumptions:
  - Users employ Gaussian codebooks with equal length codewords.
  - A simplified H-K type scheme with a fixed power split and no time-sharing.
- $\alpha \in [0, 1]$ and $\beta \in [0, 1]$: Private message power ratios of user 1 and user 2 respectively.
- $\mathcal{F}$: Achievable rate pairs $(R_1, R_2) \in \mathbb{R}_+^2$

Achievable Rate Region $\mathcal{F}$

$$R_1 \leq \phi_1 = C\left(\frac{P_1}{1+a\beta P_2}\right)$$

$$R_2 \leq \phi_2 = C\left(\frac{P_2}{1+b\alpha P_1}\right)$$

$$R_1 + R_2 \leq \phi_3 = \min\{\phi_{31}, \phi_{32}, \phi_{33}\}$$

with

$$\phi_{31} = C\left(\frac{P_1 + a(1-\beta)P_2}{1+a\beta P_2}\right) + C\left(\frac{\beta P_2}{1+b\alpha P_1}\right)$$

$$\phi_{32} = C\left(\frac{\alpha P_1}{1+a\beta P_2}\right) + C\left(\frac{P_2 + b(1-\alpha)P_1}{1+b\alpha P_1}\right)$$

$$\phi_{33} = C\left(\frac{\alpha P_1 + a(1-\beta)P_2}{1+a\beta P_2}\right) + C\left(\frac{\beta P_2 + b(1-\alpha)P_1}{1+b\alpha P_1}\right)$$

$$\begin{aligned} 2R_1 + R_2 \leq \phi_4 = \quad & C\left(\frac{P_1 + a(1-\beta)P_2}{1+a\beta P_2}\right) + C\left(\frac{\alpha P_1}{1+a\beta P_2}\right) \\ & + C\left(\frac{\beta P_2 + b(1-\alpha)P_1}{1+b\alpha P_1}\right) \end{aligned}$$

$$\begin{aligned} R_1 + 2R_2 \leq \phi_5 = \quad & C\left(\frac{P_2 + b(1-\alpha)P_1}{1+b\alpha P_1}\right) + C\left(\frac{\beta P_2}{1+b\alpha P_1}\right) \\ & + C\left(\frac{\alpha P_1 + a(1-\beta)P_2}{1+a\beta P_2}\right) \end{aligned}$$

where $C(x) = 1/2\log_2(1+x)$.

Achievable Rate Region in Matrix Form

- Denote the H-K scheme that achieves the above rate region $\mathcal{F}$ by $\text{HK}(\alpha, \beta)$.
- $\mathcal{F}$ can be represented in a matrix form as $\mathcal{F} = \{\mathbf{R} | \mathbf{R} \geq \mathbf{0},\ \mathbf{R} \leq \mathbf{R}^1, \text{ and } \mathbf{AR} \leq \mathbf{B}\}$, where $\mathbf{R} = (R_1\ R_2)^t$, $\mathbf{R}^1 = (\phi_1\ \phi_2)^t$, $\mathbf{B} = (\phi_3\ \phi_4\ \phi_5)^t$, and

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}^t$$

Bargaining Problem for Rates

- *Feasible set*:
  - Set of all possible agreements that users can jointly achieve.
    - The rate region $\mathcal{F}$ achieved by HK(α, β).
- *Disagreement point*:
  - Rate allocation that results when users fail to agree.
    - Each user treats the other's signal as noise $\mathbf{R}^0 = (C(\frac{P_1}{1+aP_2})\ C(\frac{P_2}{1+bP_1}))^t$.
- Bargaining problem represented by $(\mathcal{F}, \mathbf{R}^0)$.

Definition: $(\mathcal{F}, \mathbf{R}^0)$ is *essential* iff $\mathcal{F} \cap \{\mathbf{R} | \mathbf{R} > \mathbf{R}^0\}$ is nonempty.

Axiomatic Definition of NBS

$\mathbf{R}^* = \mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0)$ is a NBS for $(\mathcal{F}, \mathbf{R}^0)$, if the following are satisfied:

1. Individual Rationality: $\Phi_i(\mathcal{F}, \mathbf{R}^0) \geq R_i^0, \forall i$
2. Feasibility: $\mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0) \in \mathcal{F}$
3. Pareto Optimality: $\mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0)$ is Pareto optimal.
4. Independence of Irrelevant Alternatives: For any closed convex set $\mathcal{G}$, if $\mathcal{G} \subseteq \mathcal{F}$ and $\mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0) \in \mathcal{G}$, then $\mathbf{\Phi}(\mathcal{G}, \mathbf{R}^0) = \mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0)$.
5. Scale Invariance: For any numbers $\lambda_1, \lambda_2, \gamma_1$ and γ_2, such that $\lambda_1 > 0$ and $\lambda_2 > 0$, if $\mathcal{G} = \{(\lambda_1 R_1 + \gamma_1, \lambda_2 R_2 + \gamma_2) | (R_1, R_2) \in \mathcal{F}\}$ and $\mathbf{g}^0 = (\lambda_1 R_1^0 + \gamma_1, \lambda_2 R_2^0 + \gamma_2)$, then $\mathbf{\Phi}(\mathcal{G}, \mathbf{g}^0) = (\lambda_1 \Phi_1(\mathcal{F}, \mathbf{R}^0) + \gamma_1, \lambda_2 \Phi_2(\mathcal{F}, \mathbf{R}^0) + \gamma_2)$.
6. Symmetry: If $R_1^0 = R_2^0$, and $\{(R_2, R_1) | (R_1, R_2) \in \mathcal{F}\} = \mathcal{F}$, then $\Phi_1(\mathcal{F}, \mathbf{R}^0) = \Phi_2(\mathcal{F}, \mathbf{R}^0)$.

Nash Bargaining Solution

Theorem

The unique solution $\mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0)$ *satisfying all six axioms above is given by,*

$$\mathbf{\Phi}(\mathcal{F}, \mathbf{R}^0) = \arg \max_{\mathbf{R} \in \mathcal{F}, \mathbf{R} \geq \mathbf{R}^0} \prod_{i=1}^{2} (R_i - R_i^0)$$

Gaussian MAC

- The capacity region $\mathcal{C}$ is given by (R_1, R_2) such that

$$R_i \leq C(P_i),\ i \in \{1, 2\}$$
$$R_1 + R_2 \leq \phi_0 = C(P_1 + P_2)$$

- If users fully cooperate, any rate pair is achievable.
- Treating the other user's signal as noise leads to rate $R_i^0 = C(\frac{P_i}{1+P_{3-i}})$ for user i.
  - Use as the disagreement point.
- Nash bargaining problem: $(\mathcal{C}, \mathbf{R}^0)$

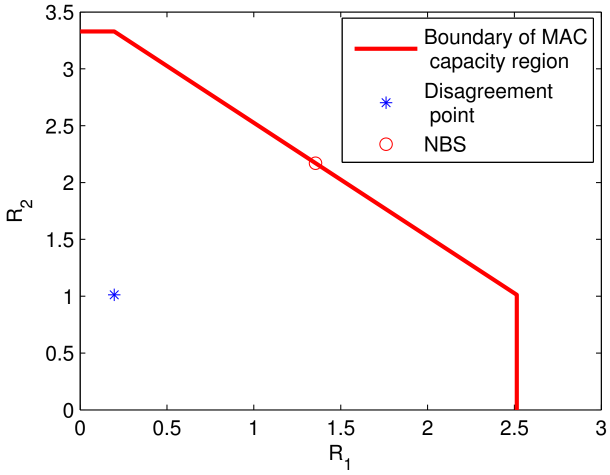
Gaussian MAC NBS

Proposition

There exists a unique NBS for the bargaining problem $(\mathcal{C}, \mathbf{R}^0)$*, given by* $\mathbf{R}^* = (R_1^0 + \frac{1}{\mu_1}, R_2^0 + \frac{1}{\mu_1})$ *where* $\mu_1 = \frac{2}{\phi_0 - R_1^0 - R_2^0}$.

Proof: Formulate the Nash optimization problem and invoke the KKT conditions.

Gaussian MAC NBS



$\mathrm{SNR}_1 = 15\mathrm{dB}$, $\mathrm{SNR}_2 = 20\mathrm{dB}$

Two-Phase Mechanism for Gaussian IC

- Phase 1:
  - Negotiate for a simple H-K scheme that has the potential to improve individual rates for both.
  - Negotiation breakdown if one user does not have an incentive to cooperate.
- Phase 2:
  - Bargain for a fair rate pair over the achievable rate region of the H-K scheme agreed on in Phase 1.

Phase 1 Incentive Conditions: Strong Interference

- $a \geq 1$ and $b \geq 1$:
  - Choose optimal $\alpha = \beta = 0$.
  - Bargaining problem $(\mathcal{F}, \mathbf{R}^0)$ is essential, both users always have incentives to cooperate.

Phase 1 Incentive Conditions: Weak Interference

- $a < 1$ and $b < 1$:
    - Use power splits $\alpha = \min(1/(bP_1), 1)$ and $\beta = \min(1/(aP_2), 1)$.
        - At most 1-bit away from the capacity [Etkin, Tse, Wang 08]
    - For $bP_1 \leq 1$, $\mathrm{HK}(1, \beta)$ doesn't improve user 2's rate
        - User 2 does not have an incentive to cooperate.
    - Both users can improve upon $\mathbf{R}^0$ and agree to cooperate only when
        - $aP_2 > 1$, $bP_1 > 1$ and $\mathcal{F} \cap \{\mathbf{R} > \mathbf{R}^0\}$ is nonempty for $\mathrm{HK}(1/(bP_1)1/(aP_2))$,

Phase 1 Incentive Conditions: Mixed Interference

- $a < 1$ and $b \geq 1$:
  - Use near-optimal power splits $\alpha = 0$ and $\beta = \min(1/(aP_2), 1)$.
  - Similar to the weak case, both users agree to cooperate if $aP_2 > 1$ and $\mathcal{F} \cap \{\mathbf{R} > \mathbf{R}^0\}$ is nonempty for $\mathrm{HK}(0, 1/(aP_2))$.

Phase 2: NBS over IC

Proposition

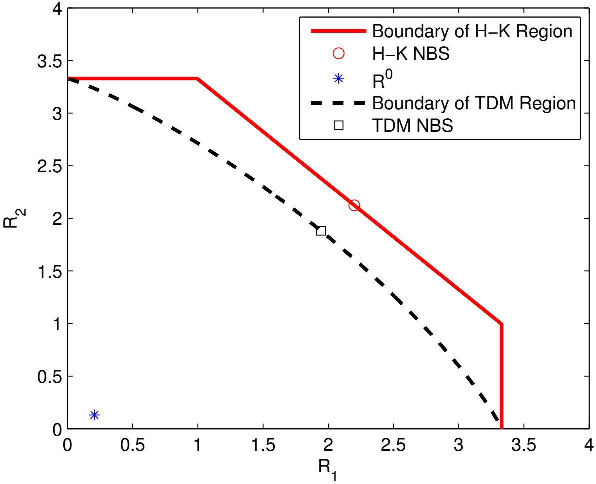
Assuming that $\mathbf{R}^0 < \mathbf{R}^1$ *and* $\mathbf{AR}^0 < \mathbf{B}$, *there exists a unique NBS* $\mathbf{R}^*$ *for the bargaining problem* $(\mathcal{F}, \mathbf{R}^0)$, *and is characterized as follows:*

$$R_i^* = \min\left\{R_i^1; R_i^0 + \frac{1}{\sum_{j=1}^3 \mu_j A_{ji}}\right\}, \quad i \in \{1,2\}$$

where $\mu_j \geq 0$, $j \in \{1,2,3\}$ *is chosen to satisfy*

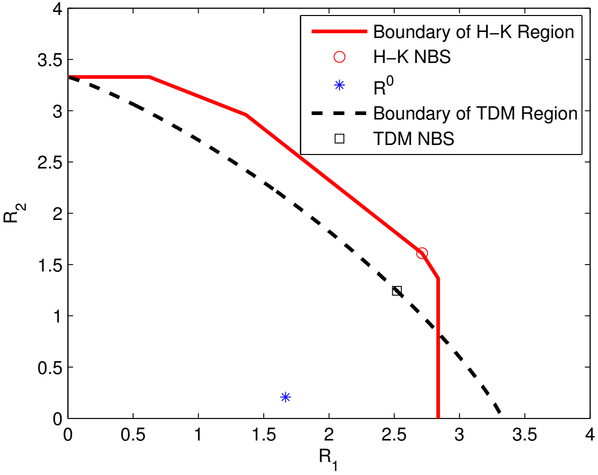
$$(\mathbf{AR}^* - \mathbf{B})_j \mu_j = 0, \quad \mathbf{AR}^* \leq \mathbf{B}$$

Strong Interference



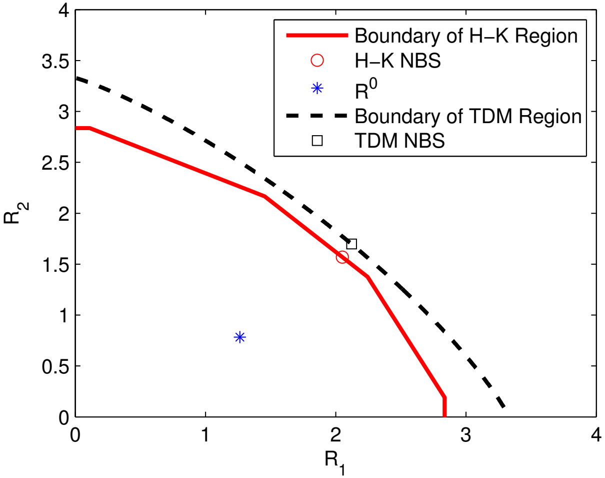
$a = 3$, $b = 5$, $\mathrm{SNR}_1 = 20\mathrm{dB}$, $\mathrm{SNR}_2 = 20\mathrm{dB}$

Mixed Interference



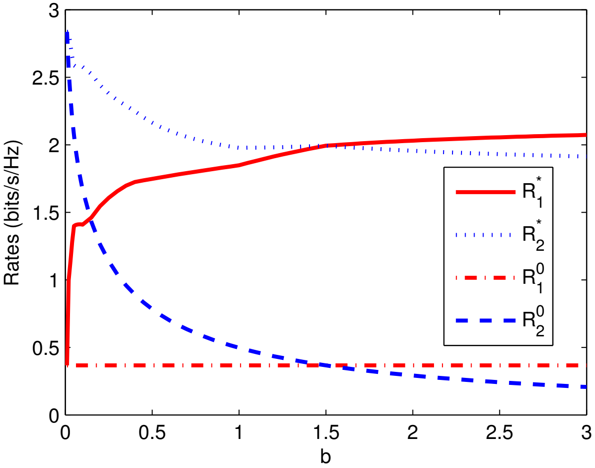
$a = 0.1$, $b = 3$, $\mathrm{SNR}_1 = 20\mathrm{dB}$, $\mathrm{SNR}_2 = 20\mathrm{dB}$

Weak Interference



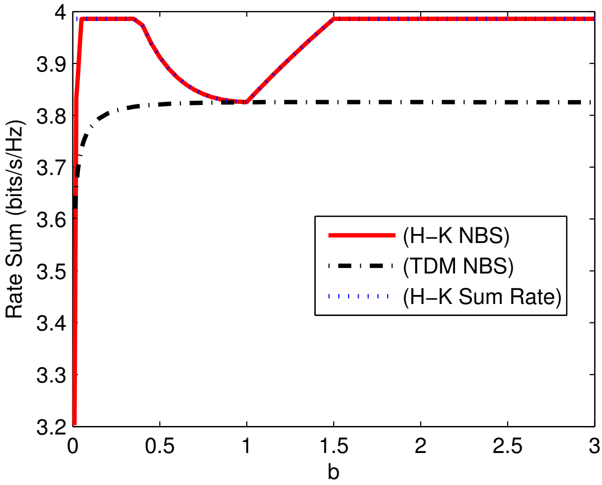
$a = 0.2$, $b = 0.5$, $SNR_1 = 20\text{dB}$, $SNR_2 = 20\text{dB}$

Rates versus Interference



NBS ($\mathbf{R}^*$) and disagreement point ($\mathbf{R}^0$), $\mathrm{SNR}_1 = \mathrm{SNR}_2 = 20\mathrm{dB}$, $a = 1.5$.

Sum Rate versus Interference



$\text{SNR}_1 = \text{SNR}_2 = 20\text{dB},\ a = 1.5.$

Some Limitations of the NBS Approach

- Most information concerning the bargaining environment and procedure is abstracted away.
- Each user's cost of delay in bargaining is not taken into account.
- We also consider the strategic approach of dynamic alternating-offer bargaining games (AOBG).

Features of the AOBG

- Model bargaining process as a noncooperative multi-stage game.
- Users alternate making offers in feasible set $\mathcal{F}$ until one is accepted.
- Cost of bargaining: An exogenous probability characterizing the risk of breakdown of bargaining due to some outside intervention.

Bargaining Procedure

- User 1 and user 2 alternate making offers.
- If user 2 rejects the offer made by user 1, there is a probability p_1 that the bargaining will end in the disagreement $\mathbf{R}^0$.
- Similarly, define p_2.
- Bargaining continues until some offer is accepted or the game ends in disagreement.
- When an offer is accepted, the users get the rates specified in the accepted offer.

Equilibrium of the AOBG

Proposition

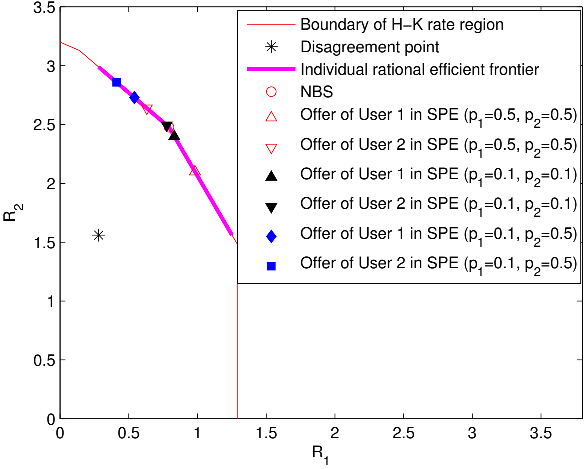
For any regular two-user bargaining problem $(\mathcal{F}, \mathbf{R}^0)$*, let* $(\bar{\mathbf{R}}, \tilde{\mathbf{R}})$ *be the unique pair of efficient agreements in* $\mathcal{F}$ *which satisfy*

$$\tilde{R}_1 = (1 - p_2)(\bar{R}_1 - R_1^0) + R_1^0$$

$$\bar{R}_2 = (1 - p_1)(\tilde{R}_2 - R_2^0) + R_2^0$$

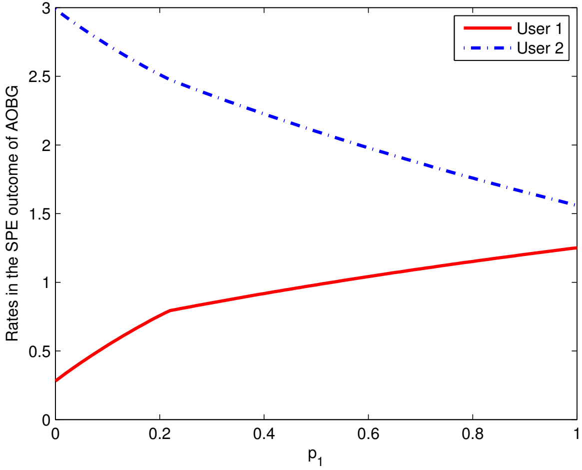
In the equilibrium, user 1 always proposes $\bar{\mathbf{R}}$ *and accepts* $\mathbf{R}$ *with* $R_1 \geq \tilde{R}_1$*; user 2 always proposes* $\tilde{\mathbf{R}}$ *and accepts* $\mathbf{R}$ *with* $R_2 \geq \bar{R}_2$*. Therefore, in equilibrium, the game will end in an agreement on* $\bar{\mathbf{R}}$ *at round 1.*

Illustration of AOBG



$a = 0.2,\ b = 1.2,\ \mathrm{SNR}_1 = 10\mathrm{dB}$ and $\mathrm{SNR}_2 = 20\mathrm{dB}$

Rate versus p_1 in AOBG



$p_2 = 0.5$, $a = 0.2$, $b = 1.2$, $\mathrm{SNR}_1 = 10\mathrm{dB}$ and $\mathrm{SNR}_2 = 20\mathrm{dB}$

Conclusion

- Coordination and bargaining can improve selfish users' rate substantially compared with the uncoordinated case.
- NBS based on a simple H-K scheme not only provides a fair operating point but also maintains a good overall performance.
- AOBG models the bargaining process.
- Cost of bargaining: Risk of bargaining breakdown.
- Ongoing and future work:
  - Bargaining for degrees of freedom.
  - Other costs of bargaining: Reduction in utility.