Information-Theoretic Considerations on Femtocells and Network MIMO

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Introduction

- Current state-of-the art transmission schemes for cellular systems are interference-limited
- Two complementary ideas:

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- Two complementary ideas:
- 1. *Femtocells:* low-power indoor transmissions

$\longrightarrow$ more users per cell

- Current state-of-the art transmission schemes for cellular systems are interference-limited
- Two complementary ideas:
- 1. *Femtocells:* low-power indoor transmissions

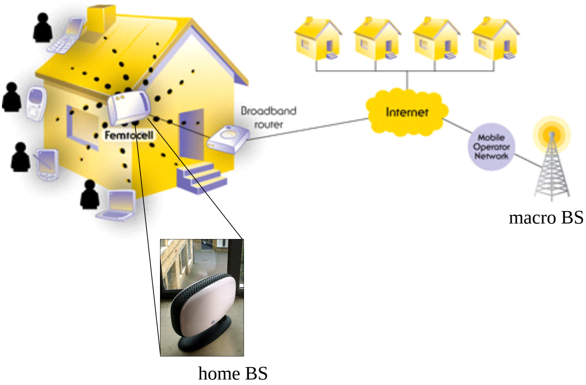
  → more users per cell 

- 2. *Network MIMO* or *multicell processing* (MCP): joint coding/ decoding across cells

  → reduced inter-cell interference 

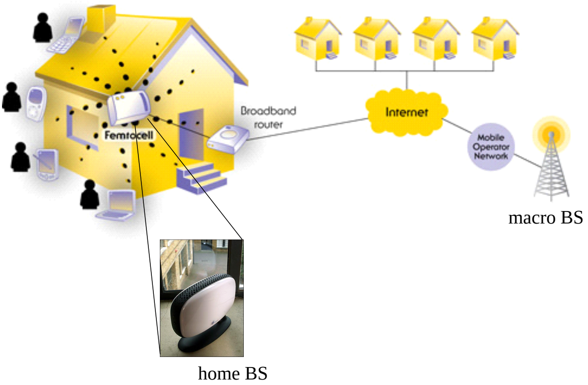
Introduction: Femtocells

- Femtocell served by a home base station (HBS)



- In-band transmission (unlike vertical handover)
- Cheap backhaul (DSL,cable + Internet)

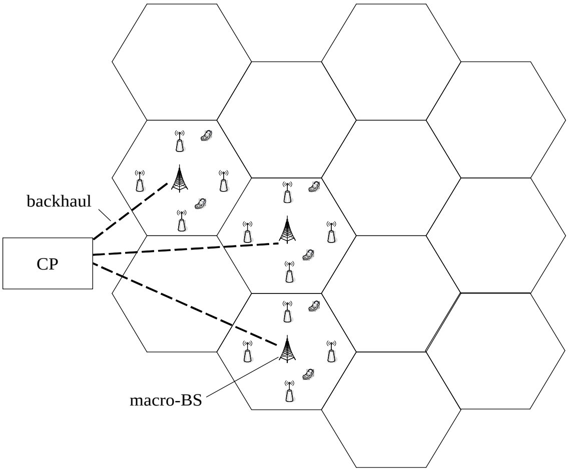
Introduction: Femtocells



- *Closed-access* HBS: Serves only indoor users
- *Open-access* HBS: Serves both outdoor and indoor users

Introduction: Network MIMO

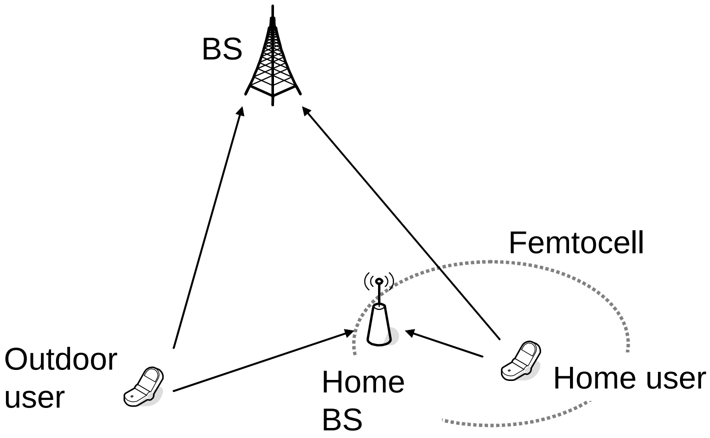
- Joint encoding/ decoding across multiple cells
- Backhaul links from BSs to some central processor



This Talk

- Information-theoretic viewpoint
- Impact of operating the HBS as an out-of-band relay
- Single-cell scenario
- Multi-cell scenario (with Network MIMO)

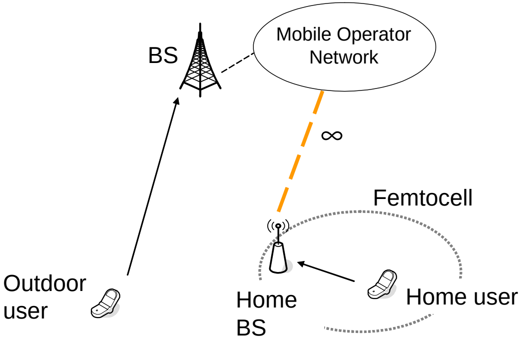
Single Cell: Scenario



- Uplink, Single femtocell

Single Cell: Conventional Assumptions

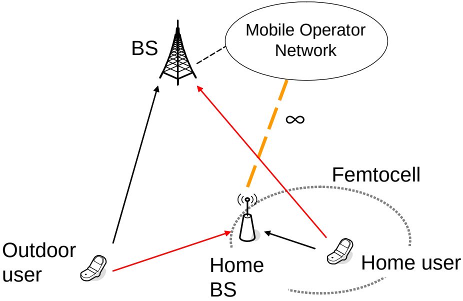
[Sezgin et al 09] [Chandrasekhar et al 10]



- outdoor user -> BS, home user-> HBS
- HBS access link infinite capacity and fully reliable

Single Cell: Conventional Assumptions

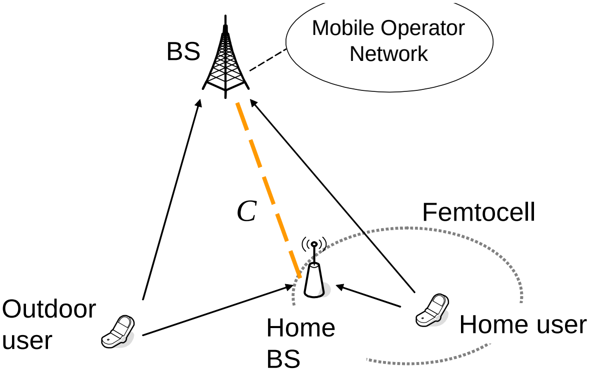
[Sezgin et al 09] [Chandrasekhar et al 10]



- outdoor user -> BS, home user-> HBS
- HBS access link infinite capacity and fully reliable

→ Home user-BS and outdoor user-HBS signals create **interference** (except with Open-Access HBS)

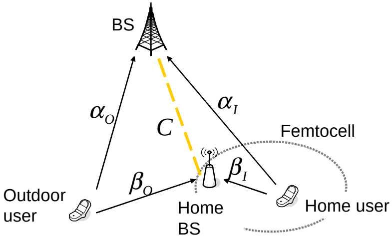
Single Cell: Our Assumptions



- **The BS decodes both users** based on received signal from users and from HBS
- HBS access link to the BS decoder
- HBS receives from the two users and **sends** C **bits/s/Hz** to the BS

Outage Analysis: System Model

- Information-theoretic model – Multiple Access Channel with Out-of-Band (Primitive) Relaying

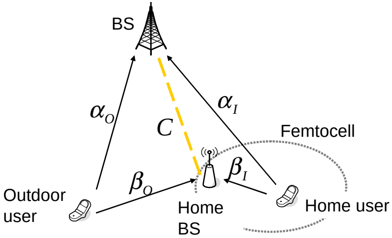


- Uplink received signal (Rayleigh fading, time sync)

$$\begin{aligned} y_{B,t} &= \sqrt{\alpha_I} h_{IB} x_{I,t} + \sqrt{\alpha_O} h_{OB} x_{O,t} + z_{B,t}, \\ y_{H,t} &= \sqrt{\beta_I} h_{IH} x_{I,t} + \sqrt{\beta_O} h_{OH} x_{O,t} + +z_{H,t}, \end{aligned}$$

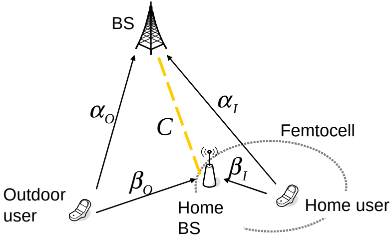
with power constraints $\frac{1}{n} \sum_{t=1}^{n} E\left[|x_{i,t}|^2\right] \leq \rho$.

Outage Analysis: Open Access vs. Closed Access



- *Closed Access* (CA):
  - The HBS *attempts* to decode home user by treating outdoor as noise
  - If HBS decoding successful, HBS sends (up to) C bits/s/Hz of home user message to BS

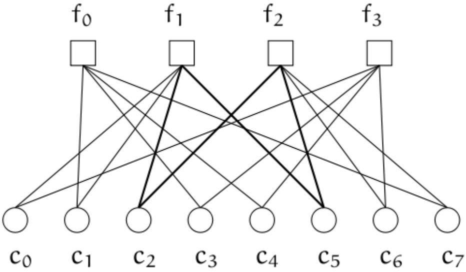
Outage Analysis: Open Access vs. Closed Access



- *Open Access* (OA):
  - The HBS attempts to decode **both** indoor and outdoor users' signals
  - If only one decoded, HBS sends (up to) C bits/s/Hz of decoded message to BS
  - If both decoded, HBS sends (up to) γC bits/s/Hz for the home user and up to $(1-\gamma)\,C$ bits/s/Hz for outdoor user $(0 \leq \gamma \leq 1)$

Outage Analysis: Partial Message Transmission

- When the BS receives b bits/s/Hz of a message of rate R, effective rate decreased to $R - b$
- Information-theory: Binning [Draper et al 03]
- In practice: Coset coding, "fixing" bits in iterative decoding for LDPC or turbo codes



Single Cell: Outage Analysis

- Fix rates R_I and R_O
- Common outage probability: *At least one* message not successfully decoded at the BS
- Open Access: Using law of total probability and extending [Narasimhan 07]

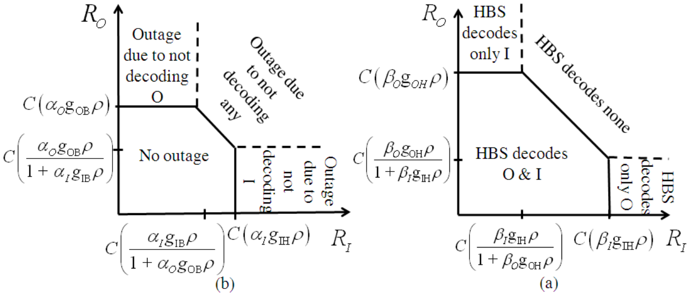
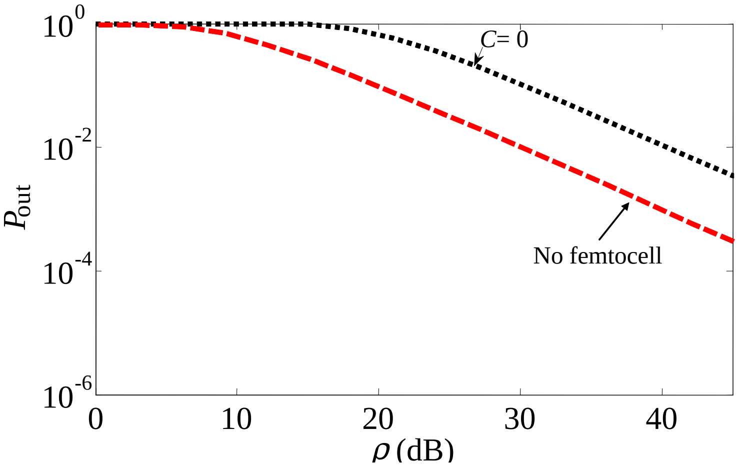


Figure:

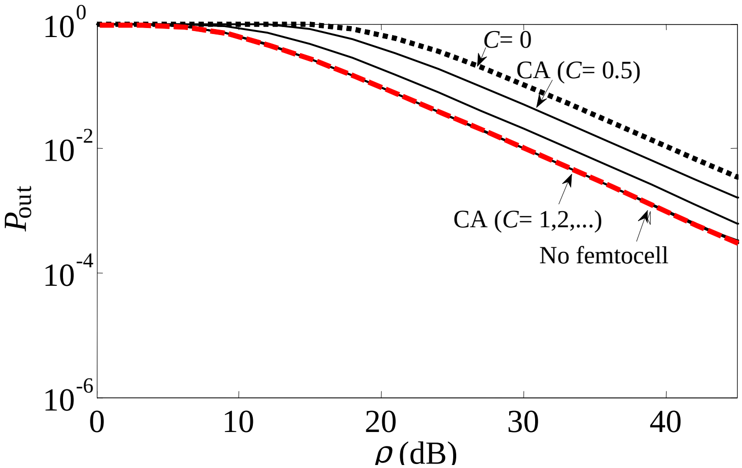
Single Cell: Outage Analysis

- $R_I = R_O = 1$, $\alpha_O = -10dB$, $\alpha_I = -20dB$, $\beta_O = 10dB$, $\beta_I = 20dB$



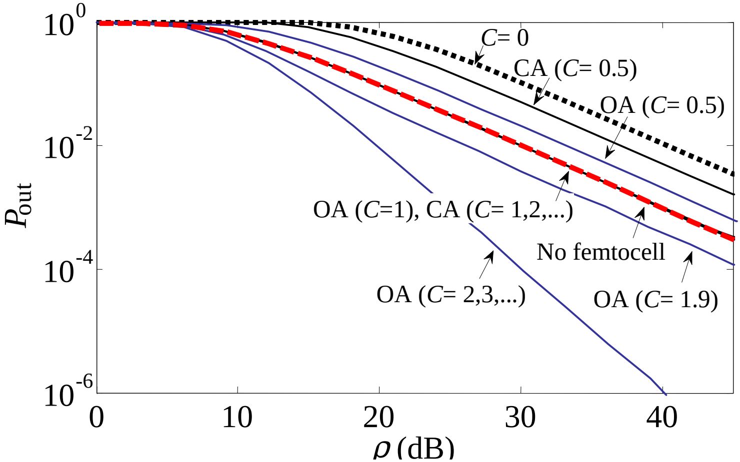
- No femtocell (NF): Only outdoor user and $C = 0$

Single Cell: Outage Analysis



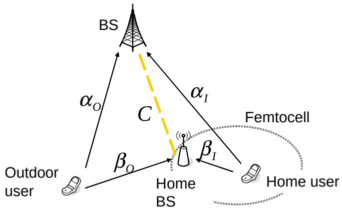
- For $C \geq 1$, CA performs as well as NF (interference cancellation at BS)

Single Cell: Outage Analysis



- OA can outperform NF! (For $C \geq R_I + R_O = 2$, increased diversity order)

Single Cell: Diversity-Multiplexing Trade-Off



- High-SNR analysis with SNR-dependent rate
- Multiplexing gain r: $R_O = r \log_2 \rho$ and $R_I = r \log_2 \rho$
- Diversity: $\lim_{\rho \to \infty} \frac{-\log P_{out}}{\log \rho} = d(r)$
- HBS-BS backhaul: $C = c \log_2 \rho$ for some $c \geq 0$
- Channel gains: $\alpha_i = \rho^{\bar{\alpha}_i - 1}$ and $\beta_i = \rho^{\bar{\beta}_i - 1}$ — $\bar{\alpha}_i$ and $\bar{\beta}_i$ scaling of $\alpha_i \rho$ and $\beta_i \rho$ in dB

Single Cell: Diversity-Multiplexing Trade-off

Theorem

The following DMT is achievable for a femtocell with CA

$$d^{CA}(r) = \min\left\{d_{out|I},\ d_{H,none} + d_{out|none}\right\},$$

where

$$\begin{aligned} d_{out|I} &= d_{out}(r,(r-c))^{+}, \\ d_{H,none} &= (\bar{\beta}_I - \bar{\beta}_O - r)^{+}, \\ \text{and } d_{out|none} &= d_{out}(r,r), \end{aligned}$$

with

$$d_{out}(r_O, r_I) = \min\{(\bar{\alpha}_I + \bar{\alpha}_O - 2(r_O + r_I))^{+}, (\bar{\alpha}_O - r_O)^{+}, (\bar{\alpha}_I - r_I)^{+}\}.$$

Single Cell: Diversity-Multiplexing Trade-off

Theorem

The following DMT is achievable for a femtocell with OA

$$d^{OA}(r) = \max_{0\leq\gamma\leq 1} \min\{d_{out|OI},\ d_{H,O} + d_{out|O},\ d_{H,I} + d_{out|I}, d_{H,none} + d_{out|none}$$

where

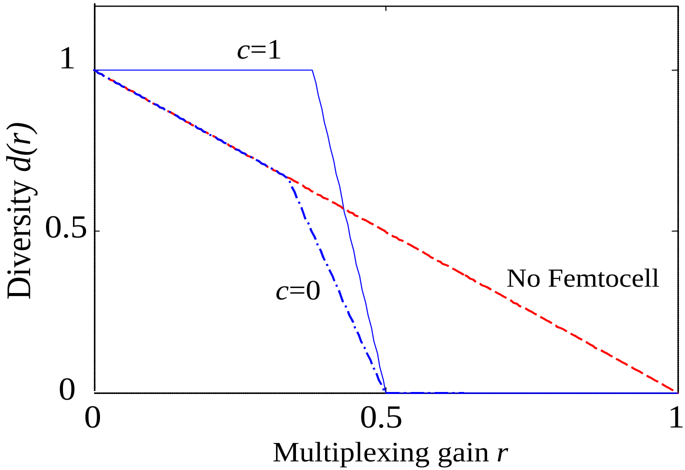
$$\begin{aligned} d_{out|OI} &= d_{out}\left((r-\gamma c)^+, (r-(1-\gamma)c)^+\right), \\ d_{out|O} &= d_{out}\left((r-c)^+, r\right), \\ d_{out|I} &= d_{out}\left(r, (r-c)^+\right), \\ d_{out|none} &= d_{out}(r, r), \end{aligned}$$

and

$$d_{H,O} = (\bar{\beta}_I - r)^+,\ d_{H,I} = (\bar{\beta}_O - r)^+,\ d_{H,none} = (\bar{\beta}_I + \bar{\beta}_O - 4r)^+.$$

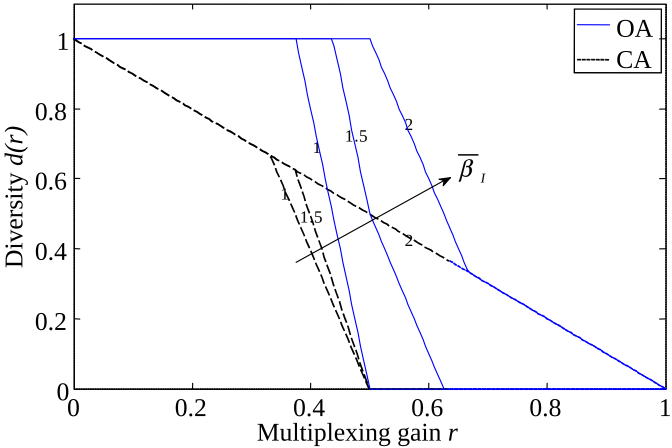
Single Cell: Diversity-Multiplexing Trade-off

- $\bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = \bar{\beta}_I = 1$

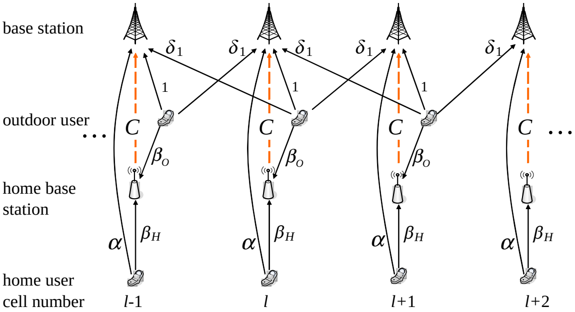


Single Cell: Diversity-Multiplexing Trade-off

- $\bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = 1$

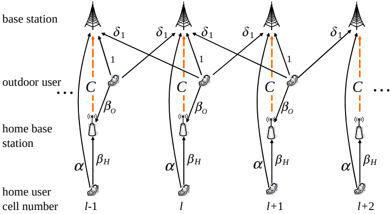


Multi Cell Scenario: System Model



- Indoor user signal received only by local HBS and BS
- Outdoor user signal received by local HBS and BS, but also L adjacent BSs on either side with symmetric power gains δ_l, $l \in [1, L]$ ($L = 1$ in figure)

Multi Cell Scenario: System Model



$$Y_l = \sum_{i=-L}^{L} \sqrt{\delta_i} X_{O,[l+i]} + \sqrt{\alpha} X_{H,l} + N_{Y,l}$$

$$\text{and } Z_l = \sqrt{\beta_O} X_{O,l} + \sqrt{\beta_H} X_{H,l} + N_{Z,l},$$

- Unit power Gaussian noise, power constraints $P_O, P_{H.}$, circulant model to avoid edge effects
- Each HBS connected to the local BS via an out-of-band link of capacity is C bits/ dim

System Model

→ Focus on home users and outdoor users rates R_H and R_O achievable in each cell

We consider:

- *Single-cell Processing* (SCP): The BS in each cell decodes independently (both indoor and outdoor users)
- *Multicell Processing* (MCP): All BSs connected to a central processor (CP) for joint decoding (of both indoor and outdoor users)

Open Access

Two classes of strategies:

- *Decode-and-Forward* (DF): As above, the HBS decodes both home and outdoor users' messages
- *Compress-and-Forward* (CF): The HBS simply compresses the received signal without decoding
- Mixed schemes with DF for indoor and CF for outdoor are also possible (not shown here)

Single-Cell Processing: Closed Access

Define $\mathcal{C}(\mathbf{A}) = \log |\mathbf{I}_k + \mathbf{A}|$ for a $k \times k$ semi-definite positive matrix $\mathbf{A}$

Theorem

(CA,SCP): Rates satisfying the following conditions

$$
\begin{aligned}
R_H &< \min\left\{\mathcal{C}\left(\frac{\beta_H P_H}{1+\beta_O P_O}\right), \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O}\right) + C\right\} \\
R_O &< \mathcal{C}\left(\frac{P_O}{1+\Delta P_O}\right) \\
R_O + R_H &< \mathcal{C}\left(\frac{P_O + \alpha P_H}{1+\Delta P_O}\right) + C,
\end{aligned}
$$

are achievable with SCP and CA femtocells, where with $\Delta = 2\sum_{l=1}^{L} \delta_l$.

Single-Cell Processing: Open Access

Theorem

(OA-DF,SCP): The convex hull of the union of the rates that satisfy

$$
\begin{aligned}
R_H &< \min\left\{\mathcal{C}\left(\beta_H P_H\right), \; \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O}\right) + \gamma C\right\} \\
R_O &< \min\{\mathcal{C}\left(\beta_O P_O\right), \; \mathcal{C}\left(\frac{P_O}{1+\Delta P_O}\right) + (1-\gamma)C\} \\
R_O + R_H &< \min\{\mathcal{C}\left(\beta_H P_H + \beta_O P_O\right), \\
& \quad \mathcal{C}\left(\frac{\alpha P_H + P_O}{1+\Delta P_O}\right) + C\}
\end{aligned}
$$

for some $0 \leq \gamma \leq 1$ is achievable with SCP and OA femtocells employing DF relaying.

Proof (sketch): The HBS decodes both messages and then allocates γC bits/dim to indoor and the rest to outdoor

Single-Cell Processing: Open Access

Theorem

(OA-CF,SCP): Rates satisfying the following conditions

$$
\begin{aligned}
R_H &< \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O} + \frac{\beta_H P_H}{1+\sigma^2}\right)\\
R_O &< \mathcal{C}\left(\frac{P_O}{1+\Delta P_O} + \frac{\beta_O P_O}{1+\sigma^2}\right)\\
R_O + R_H &< \mathcal{C}\left(\mathbf{A}\right)
\end{aligned}
$$

with $\mathbf{A} = \begin{bmatrix} \frac{P_O+\alpha P_H}{1+\Delta P_O} & \frac{\sqrt{\beta_O}P_O+\sqrt{\alpha\beta_H}P_H}{\sqrt{(1+\Delta P_O)(1+\sigma^2)}} \\ \frac{\sqrt{\beta_O}P_O+\sqrt{\alpha\beta_H}P_H}{\sqrt{(1+\Delta P_O)(1+\sigma^2)}} & \frac{\beta_H P_H+\beta_O P_O}{1+\sigma^2} \end{bmatrix}$ *are achievable with SCP and OA femtocells employing CF relaying, where*

$$\sigma^2 = \frac{\left[1+\beta_O P_O+\beta_H P_H-\frac{(\sqrt{\beta_O}P_O+\sqrt{\alpha\beta_H}P_H)^2}{P_O+\alpha P_H+\Delta P_O}\right]}{2^{2C}-1}.$$

Single-Cell Processing: Open Access – Proof

Proof (sketch):

- Each HBS compresses using Wyner-Ziv (exploiting side information at the BS)
- Each BS decodes local indoor and outdoor messages based on the received signal and the signal compressed by the HBS

Multi-Cell Processing

- Decoding of all message performed jointly at the CP
- Define the channel matrix $\mathbf{H}$ between outdoor users and the BSs as the $M \times M$ circulant matrix with first column

$$[\sqrt{\delta_0}\sqrt{\delta_1}\cdots\sqrt{\delta_{L_C}}\mathbf{0}_{L-(2L_C+1)}\sqrt{\delta_{L_C}}\sqrt{\delta_{L_C-1}}\cdots\sqrt{\delta_1}].$$

- Denote the eigenvalues of $\mathbf{H}\mathbf{H}^T$ as $\lambda_l = \left(1 + 2\sum_{l=1}^{L_C}\sqrt{\delta_l}\cos\left(\frac{2\pi}{L}l\right)\right)^2$, $l \in [0, M-1]$

Multi-Cell Processing: Closed Access

Theorem

(CA,MCP): Rates satisfying the following condition:

$$\begin{aligned} R_H &< \min\left\{\mathcal{C}\left(\frac{\beta_H P_H}{1+\beta_O P_O}\right), \ \mathcal{C}\left(\alpha P_H\right) + C\right\} \\ R_O &< \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O\right) \\ R_O + R_H &< \frac{1}{L}\sum_{l=0}^{L=1}\mathcal{C}\left(\lambda_l P_O + \alpha P_H\right) + C \end{aligned}$$

are achievable with MCP and CA femtocells.

Proof (sketch): The HBS decodes indoor treating outdoor as noise. The CP decodes jointly.

Multi-Cell Processing: Open Access

Theorem

(OA-DF,MCP): The convex hull of the union of the rates that satisfy

$$
\begin{aligned}
R_H &< \min\left\{\mathcal{C}\left(\beta_H P_H\right),\ \mathcal{C}\left(\alpha P_H\right)+\gamma C\right\} \\
R_O &< \min\left\{\begin{array}{l} \mathcal{C}\left(\beta_O P_O\right), \\ \frac{1}{L}\sum_{l=0}^{L-1}\mathcal{C}\left(\lambda_l P_O\right)+(1-\gamma)C \end{array}\right\} \\
R_O + R_H &< \min\left\{\begin{array}{l} \mathcal{C}\left(\beta_H P_H+\beta_O P_O\right), \\ \frac{1}{L}\sum_{l=0}^{L-1}\mathcal{C}\left(\lambda_l P_O+\alpha P_H\right)+C \end{array}\right\}
\end{aligned}
$$

for some $0 \leq \gamma \leq 1$ is achievable with MCP and OA femtocells employing DF relaying.

Proof (sketch): The HBS decodes both indoor and outdoor users, and the CP performs joint decoding

Multi-Cell Processing: Open Access

Theorem

(OA-CF,MCP): Rates satisfying the following conditions

$$\begin{aligned} R_H &< \mathcal{C}\left(\frac{\alpha P_H}{1+\Delta P_O} + \frac{\beta_H P_H}{1+\sigma^2}\right) \\ R_O &< \frac{1}{L}\sum_{l=0}^{L=1} \mathcal{C}\left(\lambda_l P_O + \frac{\beta_O P_O}{1+\sigma^2}\right) \\ R_O + R_H &< \frac{1}{L}\mathcal{C}\left(\mathbf{B}\right) \end{aligned}$$

with $\mathbf{B} = \begin{bmatrix} P_O \mathbf{H}\mathbf{H}^T + \alpha P_H \mathbf{I} & \frac{\sqrt{\beta_O} P_O \mathbf{H} + \sqrt{\alpha \beta_H} P_H \mathbf{I}}{\sqrt{1+\sigma^2}} \\ \frac{\sqrt{\beta_O} P_O \mathbf{H}^T + \sqrt{\alpha \beta_H} P_H \mathbf{I}}{\sqrt{1+\sigma^2}} & \left(\frac{\beta_O P_O}{1+\sigma^2} + \frac{\beta_H P_H}{1+\sigma^2}\right)\mathbf{I} \end{bmatrix}$*, are achievable with MCP and OA femtocells employing CF relaying.*

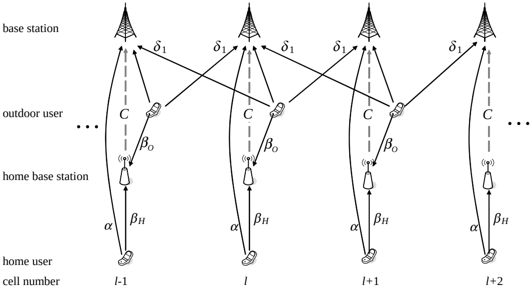
Multi-Cell Processing: Open Access – Proof

- Proof (sketch): The HBS using Wyner-Ziv quantization
- The CP decodes jointly based on all received signals and the compressed signals received from HBS
- Performance could be improved with distributed compression schemes that also exploit side information (not investigated here)

Numerical Results

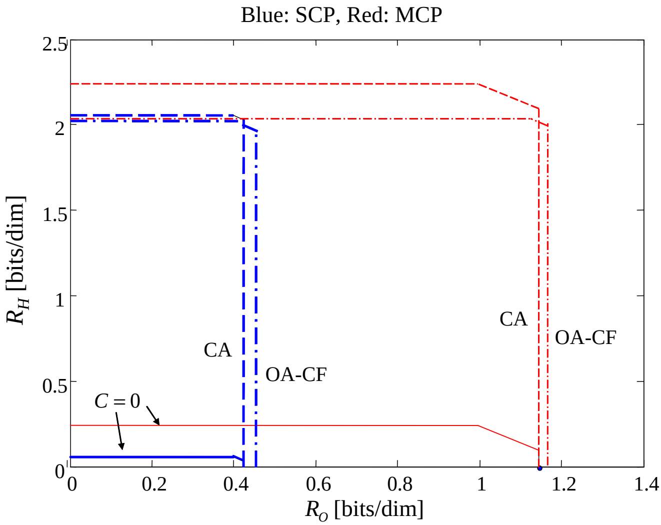
- Set $P_O = P_H = 4$, $\beta_H = 20dB$ and $\alpha = -10dB$, $M = 30$, $L = 1$

$\rightarrow$ home user-to-HBS $= 30dB$ better than home user-to-BS



Numerical Results: Rate Region – Femtocell vs. MCP

- $\delta_1 = 0.5$, $\beta_O = -3dB$ and $C = 2$

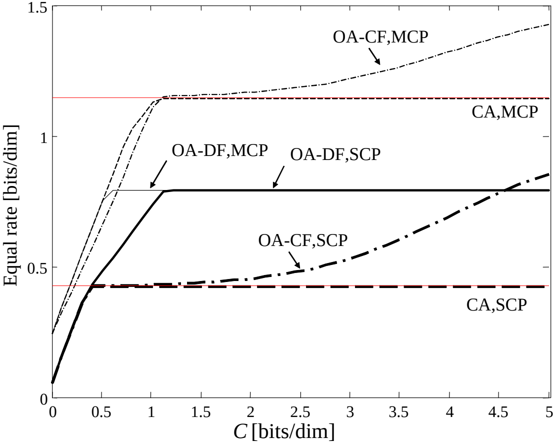


Numerical Results: Rate Region – Femtocell vs. MCP

- MCP tends to favour more outdoor users, femtocells indoor users – complementary effect
- Depending on the operating point of interest, MCP gains can be offset by the relaying gains due to HBS

Numerical Results: DF vs. CF

- $\delta_1 = 0.5$ and $\beta_O = -3dB$



Numerical Results: DF vs. CF

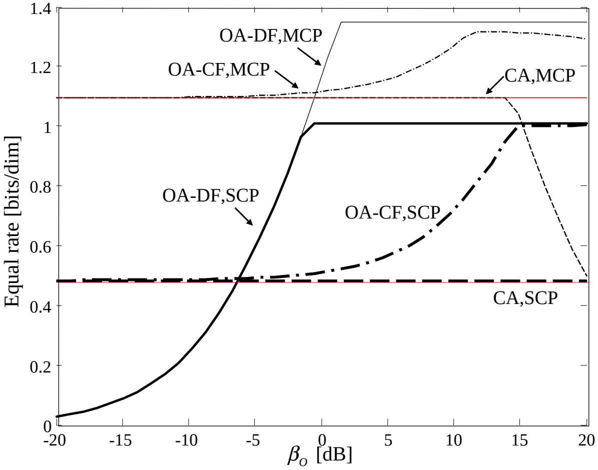
- For C small, the outdoor user is better off without femtocell
- For C small, OA-DF is appropriate (performance is limited by decoding at the BS)
- As C increases, OA-DF limited by rate decodable at the HBS and OA-CF becomes advantageous
- With MCP the crossing point between the performance of OA-DF and OA-CF occurs for smaller values of C than SCP, due to the greater decoding power at the CP

Conclusions

- Information-theoretic view
- Femtocells as out-of-band relay
- Femtocells in open-access mode
  - resilient to macro-to-femto and femto-to-macro interference
  - improve performance of both indoor and outdoor users (even without decoding capabilities)
- Network MIMO and femtocells have complementary gains

Numerical Results: CA vs. OA

- $\delta_1 = 0.4$ and $C = 1.5$

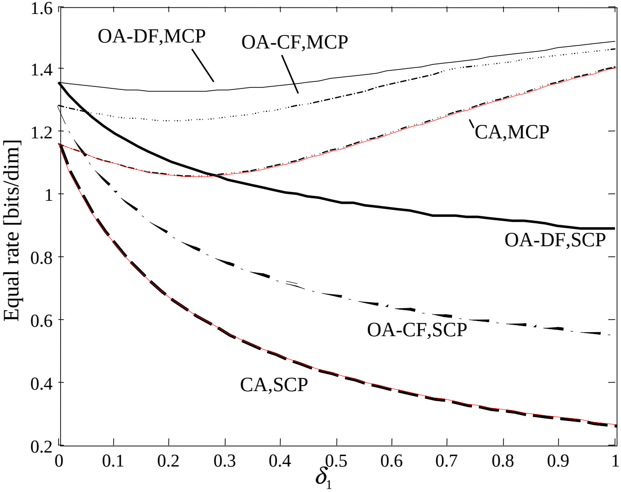


Numerical Results: CA vs. OA

- OA improves rate also of the outdoor users (with respect to no femtocell)
- OA-DF becomes advantageous over CA for sufficiently large β_O
- OA-CF, for the range of β_O shown here, performs always at least as well as CA
- OA-CF vs. OA-DF: Whenever decoding at the HBS does not set the performance bottleneck (i.e., for β_O large enough), OA-DF outperforms OA-CF, while the opposite is true when β_O is small

Numerical Results: MCP vs. SCP

- $\beta_O = 10,\ C = 1.5$



Numerical Results: MCP vs. SCP

- As the inter-cell interference δ_1 increases, advantages of MCP more pronounced
- CF performs better when deployed with MCP than with SCP