Information-Theoretic Considerations on Femtocells and Network MIMO

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Introduction

- Current state-of-the art transmission schemes for cellular systems are interference-limited
- Two complementary ideas:
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Two complementary ideas:

1. **Femtocells**: low-power indoor transmissions

→ more users per cell
Current state-of-the-art transmission schemes for cellular systems are interference-limited.

Two complementary ideas:

1. **Femtocells**: low-power indoor transmissions
   
   → more users per cell

2. **Network MIMO** or **multicell processing** (MCP): joint coding/decoding across cells
   
   → reduced inter-cell interference
Introduction: Femtocells

- Femtocell served by a home base station (HBS)

- In-band transmission (unlike vertical handover)
- Cheap backhaul (DSL, cable + Internet)
**Introduction: Femtocells**

- **Closed-access HBS**: Serves only indoor users
- **Open-access HBS**: Serves both outdoor and indoor users
Introduction: Network MIMO

- Joint encoding/decoding across multiple cells
- Backhaul links from BSs to some central processor
This Talk

- Information-theoretic viewpoint
- Impact of operating the HBS as an out-of-band relay
- Single-cell scenario
- Multi-cell scenario (with Network MIMO)
Single Cell: Scenario

- Outdoor user
- Home user
- Femtocell
- Home BS

- Uplink, Single femtocell
Single Cell: Conventional Assumptions

[Sezgin et al 09] [Chandrasekhar et al 10]

- outdoor user -> BS, home user-> HBS
- HBS access link infinite capacity and fully reliable
Single Cell: Conventional Assumptions

[Sezgin et al 09] [Chandrasekhar et al 10]

- outdoor user -> BS, home user-> HBS
- HBS access link infinite capacity and fully reliable
  → Home user-BS and outdoor user-HBS signals create **interference** (except with Open-Access HBS)
The BS decodes both users based on received signal from users and from HBS

HBS access link to the BS decoder

HBS receives from the two users and sends $C$ bits/s/Hz to the BS
Outage Analysis: System Model

- Information-theoretic model – Multiple Access Channel with Out-of-Band (Primitive) Relaying

![Diagram of a network with BS, Home user, Outdoor user, Femtocell, and their connections]

- Uplink received signal (Rayleigh fading, time sync)
  
  \[
  y_{B,t} = \sqrt{\alpha_I} h_{IB} x_{I,t} + \sqrt{\alpha_O} h_{OB} x_{O,t} + z_{B,t},
  \]
  
  \[
  y_{H,t} = \sqrt{\beta_I} h_{IH} x_{I,t} + \sqrt{\beta_O} h_{OH} x_{O,t} + z_{H,t},
  \]

  with power constraints \( \frac{1}{n} \sum_{t=1}^{n} E \left[ |x_{i,t}|^2 \right] \leq \rho \).
Closed Access (CA):
- The HBS attempts to decode home user by treating outdoor as noise
- If HBS decoding successful, HBS sends (up to) $C$ bits/s/Hz of home user message to BS
**Open Access (OA):**
- The HBS attempts to decode both indoor and outdoor users’ signals.
- If only one decoded, HBS sends (up to) $C$ bits/s/Hz of decoded message to BS.
- If both decoded, HBS sends (up to) $\gamma C$ bits/s/Hz for the home user and up to $(1 - \gamma) C$ bits/s/Hz for outdoor user ($0 \leq \gamma \leq 1$).
Outage Analysis: Partial Message Transmission

- When the BS receives $b$ bits/s/Hz of a message of rate $R$, effective rate decreased to $R - b$
- Information-theory: Binning [Draper et al 03]
- In practice: Coset coding, "fixing" bits in iterative decoding for LDPC or turbo codes
Fix rates $R_I$ and $R_O$

Common outage probability: *At least one* message not successfully decoded at the BS

Open Access: Using law of total probability and extending [Narasimhan 07]
Single Cell: Outage Analysis

- $R_I = R_O = 1$, $\alpha_O = -10 dB$, $\alpha_I = -20 dB$, $\beta_O = 10 dB$, $\beta_I = 20 dB$

- No femtocell (NF): Only outdoor user and $C = 0$
- For $C \geq 1$, CA performs as well as NF (interference cancellation at BS).
- OA can outperform NF! (For $C \geq R_I + R_O = 2$, increased diversity order)
High-SNR analysis with SNR-dependent rate

Multiplexing gain $r$: $R_O = r \log_2 \rho$ and $R_I = r \log_2 \rho$

Diversity: $\lim_{\rho \to \infty} \frac{-\log P_{out}}{\log \rho} = d(r)$

HBS-BS backhaul: $C = c \log_2 \rho$ for some $c \geq 0$

Channel gains: $\alpha_i = \rho^{\bar{\alpha}_i - 1}$ and $\beta_i = \rho^{\bar{\beta}_i - 1}$ — $\bar{\alpha}_i$ and $\bar{\beta}_i$ scaling of $\alpha_i \rho$ and $\beta_i \rho$ in dB
Theorem

The following DMT is achievable for a femtocell with CA

\[ d^{CA}(r) = \min \{ d_{out|I}, d_{H,none} + d_{out|none} \}, \]

where

\[ d_{out|I} = d_{out}(r, (r-c))^+, \]
\[ d_{H,none} = (\bar{\beta}_I - \bar{\beta}_O - r)^+, \]
\[ and \ d_{out|none} = d_{out}(r, r), \]

with

\[ d_{out}(r_O, r_I) = \min \{ (\bar{\alpha}_I + \bar{\alpha}_O - 2(r_O + r_I))^+, (\bar{\alpha}_O - r_O)^+, (\bar{\alpha}_I - r_I)^+ \}. \]
The following DMT is achievable for a femtocell with OA

\[ d^{OA}(r) = \max_{0 \leq \gamma \leq 1} \min \{ d_{out|O1}, d_{H,O} + d_{out|O}, d_{H,I} + d_{out|I}, d_{H,\text{none}} + d_{out|\text{none}} \} \]

where

\[ d_{out|O1} = d_{out} \left( (r - \gamma c)^+, (r - (1 - \gamma) c)^+ \right) , \]
\[ d_{out|O} = d_{out} \left( (r - c)^+, r \right) , \]
\[ d_{out|I} = d_{out} \left( r, (r - c)^+ \right) , \]
\[ d_{out|\text{none}} = d_{out}(r, r) , \]

and

\[ d_{H,O} = (\tilde{\beta}_I - r)^+, \quad d_{H,I} = (\tilde{\beta}_O - r)^+, \quad d_{H,\text{none}} = (\tilde{\beta}_I + \tilde{\beta}_O - 4r)^+ . \]
\( \bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = \bar{\beta}_I = 1 \)

![Graph showing diversity-multiplexing trade-off](image_url)
\[ \bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = 1 \]
Multi Cell Scenario: System Model

- Indoor user signal received only by local HBS and BS
- Outdoor user signal received by local HBS and BS, but also $L$ adjacent BSs on either side with symmetric power gains $\delta_l, l \in [1, L]$ ($L = 1$ in figure)
Multi Cell Scenario: System Model

\[
Y_l = \sum_{i=-L}^{L} \sqrt{\delta_i} X_{O,|l+i|} + \sqrt{\alpha} X_{H,l} + N_{Y,l}
\]

and
\[
Z_l = \sqrt{\beta_o} X_{O,l} + \sqrt{\beta_H} X_{H,l} + N_{Z,l},
\]

- Unit power Gaussian noise, power constraints \( P_O, P_H \), circulant model to avoid edge effects
- Each HBS connected to the local BS via an out-of-band link of capacity is \( C \) bits/ dim
Focus on home users and outdoor users rates $R_H$ and $R_O$ achievable in each cell

We consider:

- **Single-cell Processing** (SCP): The BS in each cell decodes independently (both indoor and outdoor users)
- **Multicell Processing** (MCP): All BSs connected to a central processor (CP) for joint decoding (of both indoor and outdoor users)
Two classes of strategies:

- **Decode-and-Forward (DF):** As above, the HBS decodes both home and outdoor users’ messages.
- **Compress-and-Forward (CF):** The HBS simply compresses the received signal without decoding.
- Mixed schemes with DF for indoor and CF for outdoor are also possible (not shown here).
Define $C(A) = \log |I_k + A|$ for a $k \times k$ semi-definite positive matrix $A$.

**Theorem**

$(CA,SCP)$: Rates satisfying the following conditions

\[
R_H < \min \left\{ C \left( \frac{\beta H P_H}{1 + \beta O P_O} \right), C \left( \frac{\alpha P_H}{1 + \Delta P_O} \right) + C \right\}
\]

\[
R_O < C \left( \frac{P_O}{1 + \Delta P_O} \right)
\]

\[
R_O + R_H < C \left( \frac{P_O + \alpha P_H}{1 + \Delta P_O} \right) + C,
\]

are achievable with SCP and CA femtocells, where with $\Delta = 2 \sum_{l=1}^{L} \delta_l$. 
Theorem

\((\text{OA-DF,SCP})\): The convex hull of the union of the rates that satisfy

\[
R_H < \min \left\{ \mathcal{C} (\beta_H P_H), \mathcal{C} \left( \frac{\alpha P_H}{1 + \Delta P_O} \right) + \gamma C \right\}
\]

\[
R_O < \min \{ \mathcal{C} (\beta_O P_O), \mathcal{C} \left( \frac{P_O}{1 + \Delta P_O} \right) + (1 - \gamma) C \}
\]

\[
R_O + R_H < \min \{ \mathcal{C} (\beta_H P_H + \beta_O P_O), \mathcal{C} \left( \frac{\alpha P_H + P_O}{1 + \Delta P_O} \right) + C \}
\]

for some \(0 \leq \gamma \leq 1\) is achievable with SCP and OA femtocells employing DF relaying.

Proof (sketch): The HBS decodes both messages and then allocates \(\gamma C\) bits/dim to indoor and the rest to outdoor...
Theorem

\((OA-CF,SCP)\): Rates satisfying the following conditions

\[
R_H < C \left( \frac{\alpha P_H}{1 + \Delta P_O} + \frac{\beta_H P_H}{1 + \sigma^2} \right)
\]

\[
R_O < C \left( \frac{P_O}{1 + \Delta P_O} + \frac{\beta_O P_O}{1 + \sigma^2} \right)
\]

\[
R_O + R_H < C \left( A \right)
\]

with \( A = \)

\[
\begin{bmatrix}
\frac{P_O + \alpha P_H}{1 + \Delta P_O} & \frac{\sqrt{\beta_O P_O + \alpha \beta_H P_H}}{\sqrt{(1 + \Delta P_O)(1 + \sigma^2)}} \\
\frac{\sqrt{\beta_O P_O + \alpha \beta_H P_H}}{\sqrt{(1 + \Delta P_O)(1 + \sigma^2)}} & \frac{\beta_H P_H + \beta_O P_O}{1 + \sigma^2}
\end{bmatrix}
\]

are achievable with \(SCP\) and \(OA\) femtocells employing \(CF\) relaying, where

\[
\sigma^2 = \left( 1 + \beta_O P_O + \beta_H P_H - \frac{(\sqrt{\beta_O P_O + \alpha \beta_H P_H})^2}{P_O + \alpha P_H + \Delta P_O} \right) \cdot 2^{2^C - 1}
\]
**Proof (sketch):**

- Each HBS compresses using Wyner-Ziv (exploiting side information at the BS)
- Each BS decodes local indoor and outdoor messages based on the received signal and the signal compressed by the HBS
Decoding of all message performed jointly at the CP

Define the channel matrix $\mathbf{H}$ between outdoor users and the BSs as the $M \times M$ circulant matrix with first column

$$[\sqrt{\delta_0} \sqrt{\delta_1} \cdots \sqrt{\delta_{LC}} 0_{L-(2LC+1)} \sqrt{\delta_{LC}} \sqrt{\delta_{LC-1}} \cdots \sqrt{\delta_1}].$$

Denote the eigenvalues of $\mathbf{HH}^T$ as

$$\lambda_I = \left(1 + 2 \sum_{i=1}^{LC} \sqrt{\delta_i} \cos \left(\frac{2\pi}{L} i\right)\right)^2, \quad I \in [0, M-1]$$
Theorem

\((CA,MCP)\): Rates satisfying the following condition:

\[
R_H < \min \left\{ C \left( \frac{\beta_H P_H}{1 + \beta_O P_O} \right), \quad C (\alpha P_H) + C \right\}
\]

\[
R_O < \frac{1}{L} \sum_{l=0}^{L-1} C (\lambda_l P_O)
\]

\[
R_O + R_H < \frac{1}{L} \sum_{l=0}^{L-1} C (\lambda_l P_O + \alpha P_H) + C
\]

are achievable with MCP and CA femtocells.

Proof (sketch): The HBS decodes indoor treating outdoor as noise. The CP decodes jointly.
Theorem

\((OA-DF, MCP)\): The convex hull of the union of the rates that satisfy

\[
R_H < \min \left\{ C(\beta_H P_H), \ C(\alpha P_H) + \gamma C \right\}
\]

\[
R_O < \min \left\{ \frac{1}{L} \sum_{l=0}^{L-1} C(\lambda_l P_O) + (1 - \gamma) C \right\}
\]

\[
R_O + R_H < \min \left\{ \frac{1}{L} \sum_{l=0}^{L-1} C(\lambda_l P_O + \alpha P_H) + C \right\}
\]

for some \(0 \leq \gamma \leq 1\) is achievable with MCP and OA femtocells employing DF relaying.

Proof (sketch): The HBS decodes both indoor and outdoor users, and the CP performs joint decoding.
Multi-Cell Processing: Open Access

**Theorem**

*(OA-CF, MCP)*: Rates satisfying the following conditions

\[
R_H < C \left( \frac{\alpha P_H}{1 + \Delta P_O} + \frac{\beta_H P_H}{1 + \sigma^2} \right)
\]

\[
R_O < \frac{1}{L} \sum_{l=0}^{L=1} C \left( \lambda_l P_O + \frac{\beta_O P_O}{1 + \sigma^2} \right)
\]

\[
R_O + R_H < \frac{1}{L} C (B)
\]

with \( B = \begin{bmatrix}
    P_O H H^T + \alpha P_H I & \frac{\sqrt{\beta_O P_O H} + \sqrt{\alpha^2 \beta_H P_H}}{\sqrt{1 + \sigma^2}} \\
    \frac{\sqrt{\beta_O P_O H} + \sqrt{\alpha^2 \beta_H P_H}}{\sqrt{1 + \sigma^2}} & \left( \frac{\beta_O P_O}{1 + \sigma^2} + \frac{\beta_H P_H}{1 + \sigma^2} \right) I
\end{bmatrix} \), are achievable

with MCP and OA femtocells employing CF relaying.
Proof (sketch): The HBS using Wyner-Ziv quantization
The CP decodes jointly based on all received signals and the compressed signals received from HBS
Performance could be improved with distributed compression schemes that also exploit side information (not investigated here)
Set $P_O = P_H = 4$, $\beta_H = 20dB$ and $\alpha = -10dB$, $M = 30$, $L = 1$

$\rightarrow$ home user-to-HBS $= 30dB$ better than home user-to-BS
Numerical Results: Rate Region – Femtocell vs. MCP

\[ \delta_1 = 0.5, \beta_O = -3dB \text{ and } C = 2 \]
MCP tends to favour more outdoor users, femtocells indoor users – complementary effect.

Depending on the operating point of interest, MCP gains can be offset by the relaying gains due to HBS.
Numerical Results: DF vs. CF

- $\delta_1 = 0.5$ and $\beta_O = -3dB$
Numerical Results: DF vs. CF

- For $C$ small, the outdoor user is better off without femtocell.
- For $C$ small, OA-DF is appropriate (performance is limited by decoding at the BS).
- As $C$ increases, OA-DF limited by rate decodable at the HBS and OA-CF becomes advantageous.
- With MCP the crossing point between the performance of OA-DF and OA-CF occurs for smaller values of $C$ than SCP, due to the greater decoding power at the CP.
Conclusions

- Information-theoretic view
- Femtocells as out-of-band relay
- Femtocells in open-access mode
  - resilient to macro-to-femto and femto-to-macro interference
  - improve performance of both indoor and outdoor users (even without decoding capabilities)
- Network MIMO and femtocells have complementary gains
Numerical Results: CA vs. OA

- $\delta_1 = 0.4$ and $C = 1.5$
Numerical Results: CA vs. OA

- OA improves rate also of the outdoor users (with respect to no femtocell)
- OA-DF becomes advantageous over CA for sufficiently large $\beta_O$
- OA-CF, for the range of $\beta_O$ shown here, performs always at least as well as CA
- OA-CF vs. OA-DF: Whenever decoding at the HBS does not set the performance bottleneck (i.e., for $\beta_O$ large enough), OA-DF outperforms OA-CF, while the opposite is true when $\beta_O$ is small
Numerical Results: MCP vs. SCP

- $\beta_O = 10, \ C = 1.5$
As the inter-cell interference $\delta_1$ increases, advantages of MCP more pronounced

CF performs better when deployed with MCP than with SCP