Beyond LTE: *Hundreds* of Base Station Antennas!

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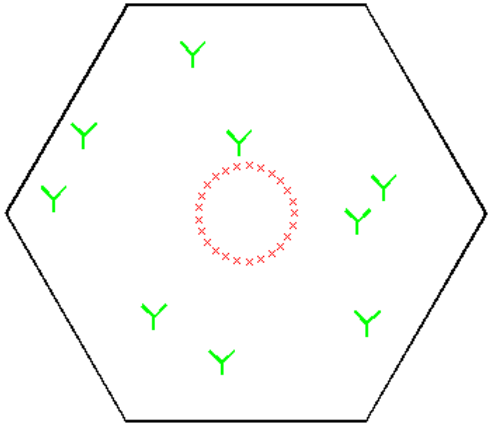
May 10, 2010

Large Excess of Base Station Antennas Over Terminals Yields Energy Efficiency + Reliably High Throughput



- M~400 base station antennas serve K~40 terminals via multi-user MIMO
- Doubling M doubles the power that the terminals receive (*given perfect CSI)*
  - **allows transmitted power to be reduced correspondingly**
  - requires base station to know forward channel
- Extra base station antennas *always* help *(even with noisy CSI)*
  - **Eventually produce inter-cellular interference-limited operation: everybody can now reduce power!**
  - eliminate effects of uncorrelated noise and fast fading
  - compensate for poor-quality channel-state information

Single Isolated Cell

[Marzetta, “How much training is required for multiuser MIMO?”, *Asilomar, 2006*]

Isolated Cell: M Antennas, K Terminals

$$\overbrace{\bar{x}}^{K\times 1} = \sqrt{\rho_{\mathrm{f}}}\;\underbrace{H}_{K\times M}\;\overbrace{\bar{s}}^{M\times 1} + \bar{w}$$

- **Perfect CSI**

$$\bar{s} = \frac{1}{\sqrt{MK}} H^{*} \bar{q} \quad \Rightarrow \quad \bar{x} = \sqrt{\frac{\rho_{\mathrm{f}}}{\mathrm{MK}}}\, HH^{*} \bar{q} + \bar{w}$$

$$x_k = \sqrt{\frac{\rho_{\mathrm{f}}}{\mathrm{MK}}}\, \bar{h}_k \bar{h}_k^{*} q_k + \left(w_k + \sqrt{\frac{\rho_{\mathrm{f}}}{\mathrm{MK}}} \sum_{\ell \neq k} \bar{h}_k \bar{h}_\ell^{*} q_\ell \right)$$

Assume that $\frac{\bar{h}_k \bar{h}_\ell^{*}}{M} \xrightarrow{M \to \infty} \delta_{k\ell}$: intra-cell interference vanishes!

iid Rayleigh → $\mathrm{SINR} \approx \dfrac{\left(\frac{M\rho_{\mathrm{f}}}{K}\right)}{1 + \frac{\rho_{\mathrm{f}}\,(K-1)}{K}}$

Isolated Cell: M Antennas, K Terminals

- **CSI estimated from reverse-link pilots**

$$\hat{H} = H + \frac{1}{\sqrt{\rho_\mathrm{r}}} V, \quad \bar{s} = \frac{1}{\sqrt{MK(1+1/\rho_\mathrm{r})}} \hat{H}^{*} \bar{q}$$

$$\bar{x} = \sqrt{\frac{\rho_\mathrm{f}\,\rho_\mathrm{r}}{\mathrm{MK}(1+\rho_\mathrm{r})}}\, H\left(H^{*} + \frac{1}{\sqrt{\rho_\mathrm{r}}} V^{*} \right)\bar{q} + \bar{w}$$

$$x_k = \sqrt{\frac{\rho_\mathrm{f}\,\rho_\mathrm{r}}{\mathrm{MK}(1+\rho_\mathrm{r})}}\, \bar{h}_k \bar{h}_k^{*} q_k$$

$$+ \left[w_k + \sqrt{\frac{\rho_\mathrm{f}}{\mathrm{MK}(1+\rho_\mathrm{r})}} \left(\sum_{\ell \neq k} \bar{h}_k \bar{h}_\ell^{*} q_\ell + \sum_{\ell=1}^{K} \bar{h}_k \bar{v}_\ell^{*} q_\ell \right) \right]$$

$$\text{iid Rayleigh} \rightarrow \quad \mathrm{SINR} \approx \frac{\left(\frac{M\rho_\mathrm{f}\,\rho_\mathrm{r}}{K} \right)}{1 + \rho_\mathrm{f} + \rho_\mathrm{r} + \frac{\rho_\mathrm{f}\,\rho_\mathrm{r}(K-1)}{(1+\rho_\mathrm{r})K}}$$

Single Isolated Cell: What Have We Learned?

- Under all reasonable propagation conditions, increasing the number of base station antennas permits a reduction in base station power
- Assign each terminal an orthogonal pilot sequence: no need to over-train
- Multiplexing gains are assured provided the cross-correlations between different channel vectors grow at a lesser rate than M:

$$\frac{\bar{h}_k \bar{h}_\ell^*}{M} \xrightarrow{M \to \infty} \delta_{k\ell}$$

Multiple Cells: No Cooperation

[Marzetta, "The ultimate performance of noncooperative cellular multiuser MIMO", *submitted to IEEE Trans. Wireless Communications*, July, 2009]

Multiple Cells: No Cooperation

- If we could assign an orthogonal pilot sequence to every terminal in every cell then nothing bad would happen!
  - Ever greater numbers of base station antennas would eventually defeat all noise, and eliminate both intra- and inter-cell interference
- But there aren't enough orthogonal pilot sequences for everyone!
  - Pilot sequences have to be re-used
- Pilot contamination: the base station inadvertently learns the channel to mobiles in other cells
  - Forward link: base station transmits interference to mobiles in other cells
  - Reverse link: base station processing enhances his reception of transmission from mobiles in other cells
- Inter-cell interference due to pilot contamination persists, even with an infinite number of antennas!
  - This is the *only* remaining impairment

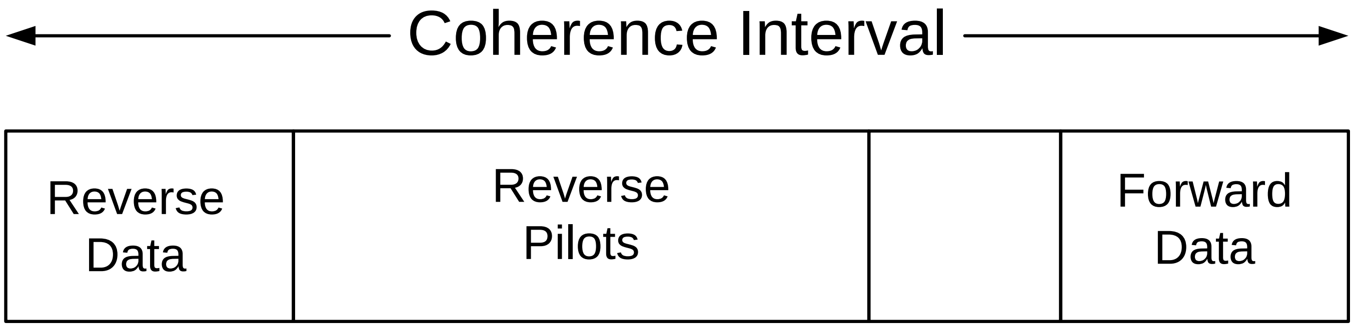
Limiting Case: Infinite Number of Antennas

- Greatly simplifies multi-cellular analysis: all effects accounted for near-analytically
  - Acquisition of CSI
  - Imperfections in CSI
  - Inter-cellular interference
  - Propagation
    - Fast (either line-of-sight, or independent Rayleigh, or something intermediate)
    - Slow (geometric, log-normal shadow)
- Far-reaching and comprehensive conclusions ensue
- Indicates a new direction in which the macro-cellular world can go: **vastly improved energy efficiency** and throughput compared with LTE

Summary of Limit Analysis

- Multi-cellular TDD scenario, 42 terminals served per cell
  - 500 μsec coherence interval (7 OFDM symbols): 3 reverse-link pilots, 1 idle, 3 data
  - OFDM: 20 MHz bandwidth, cyclic prefix 4.76 μsec
  - Fading: Fast + log-normal shadow (8 dB) + geometric (3.8 power)
  - No inter-cellular cooperation
- Net downlink throughput (comparable uplink) for frequency re-use 7
  - mean
    - 730 Mbits/sec/cell
    - 17 Mbits/sec/terminal
  - 95% likely: 3.6 Mbits/sec/terminal
  - spectral efficiency constant with respect to bandwidth
  - throughput constant with respect to cell-size
  - number of terminals per cell proportional to coherence interval
  - **performance independent of power**

Cells Operate Independently, Each Serving Single-Antenna Terminals via Multi-User MIMO: TDD Only!



Coherence Interval

Reverse Data

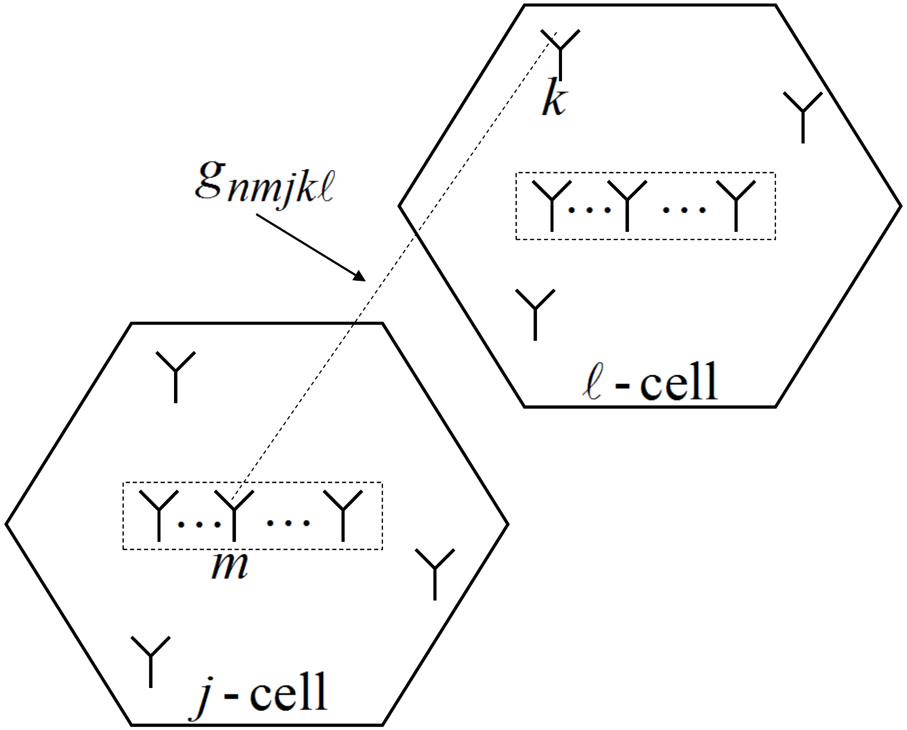
Reverse Pilots

Forward Data

- Maximum number of terminals limited by the time that it takes to send reverse pilots: pilot-interval divided by the channel delay-spread
- Coherence interval: 500 μsec (7 LTE OFDM symbols) – TGV speeds!
  - 3 symbols for reverse-link pilots
  - 3 symbols for data
  - 1 symbol for computations and dead time
- 42 terminals per cell served simultaneously

Propagation: Fast + log-normal shadow (8 dB) + geometric (3.8 power)

Nobody has prior CSI!



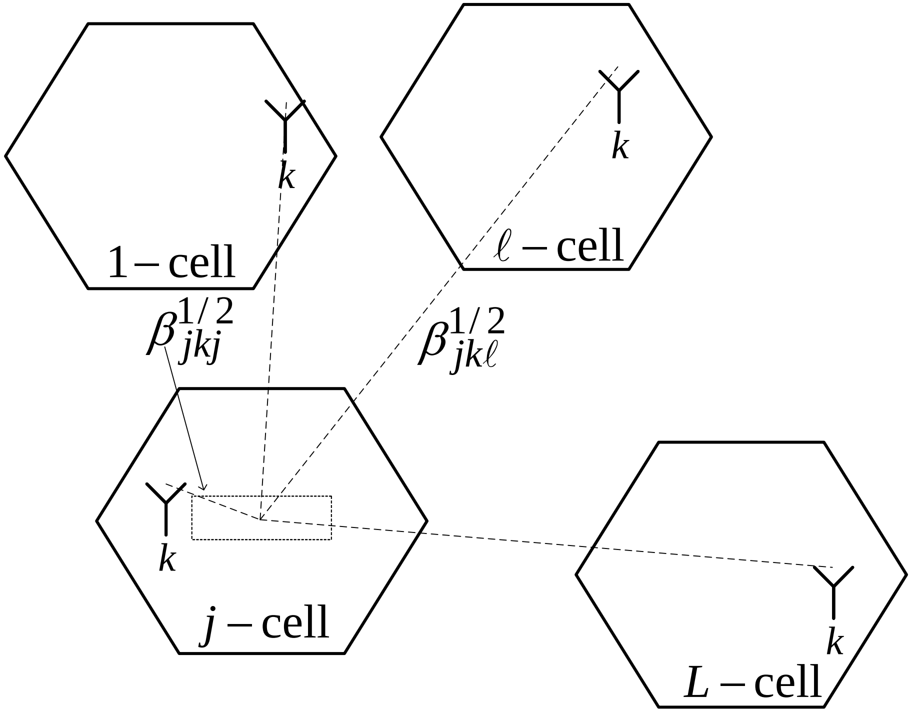
n = tone-index

$$g_{nmjk\ell} = \overbrace{h_{nmjk\ell}}^{\text{fast}} \cdot \beta_{jk\ell}^{1/2}$$

$$\beta_{jk\ell} = \frac{\overbrace{z_{jk\ell}}^{\text{shadow}}}{\underbrace{r_{jk\ell}^{\gamma}}_{\text{geometric}}}$$

Independent Rayleigh fading isn't necessary.

Pilot Contamination: Re-Use of Pilots in Other Cells Causes Base Station Inadvertently to Learn the Channel to Mobiles in Other Cells



- Pilot contamination causes inter-cellular interference on both the forward and the reverse links
- Interference persists, even for an infinite number of antennas!

Reverse-Link Pilots: Pilot Contamination

L cells share the same set of K orthogonal pilot sequences

The *j*-th base station’s estimate for the channel to his *k*-th terminal is contaminated by the channel from the terminals in *L-1* other cells who share the same pilot sequence; after de-spreading:

$$\hat{g}_{nmjkj} = g_{nmjkj} + \sum_{\ell \neq j} g_{nmjk\ell} + \frac{1}{\sqrt{\rho_{\mathrm{p}}}} v_{nmj}$$

$$\hat{G}_{jj} = G_{jj} + \sum_{\ell \neq j} G_{j\ell} + \frac{1}{\sqrt{\rho_{\mathrm{p}}}} V_j \quad (M \times K)$$

Forward-Link Data

The j-th base station uses the complex-conjugate of his channel estimate as a linear pre-coder:

$$\bar{s}_j = \hat{G}^*_{jj}\bar{a}_j$$

The terminals in the l-th cell receive transmissions from all base stations; products of identical propagation vectors grow as M, while all other products grow at a lesser rate (conclusion holds under more general conditions than independent Rayleigh fading):

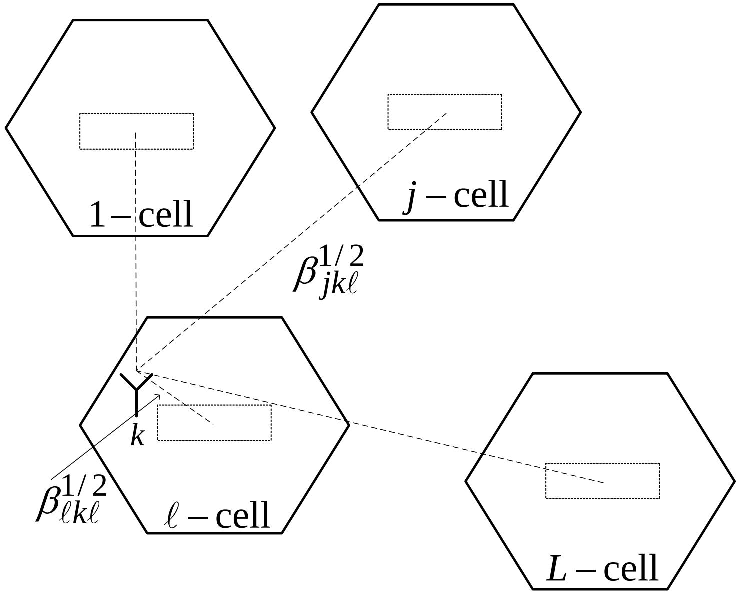
$$\bar{x}_\ell = \sqrt{\rho_f}\sum_j G^T_{j\ell}\left(\sum_n \sqrt{\rho_p}\, G^*_{jn} + V^*_j\right)\bar{a}_j + \bar{w}_\ell$$

$$G_{j\ell} = H_{j\ell} D^{1/2}_{\beta_{j\ell}}$$

$$\frac{\bar{x}_\ell}{M\sqrt{\rho_f \rho_p}} \xrightarrow{M\to\infty} D_{\bar{\beta}_{\ell\ell}}\bar{a}_\ell + \sum_{j\neq\ell} D_{\bar{\beta}_{j\ell}}\bar{a}_j, \qquad \mathrm{SIR}_{fk} = \frac{\beta^2_{\ell k\ell}}{\sum_{j\neq\ell}\beta^2_{jk\ell}}$$

Forward-Link Data: Only Remaining Impairment is Interference Due to Pilot Contamination

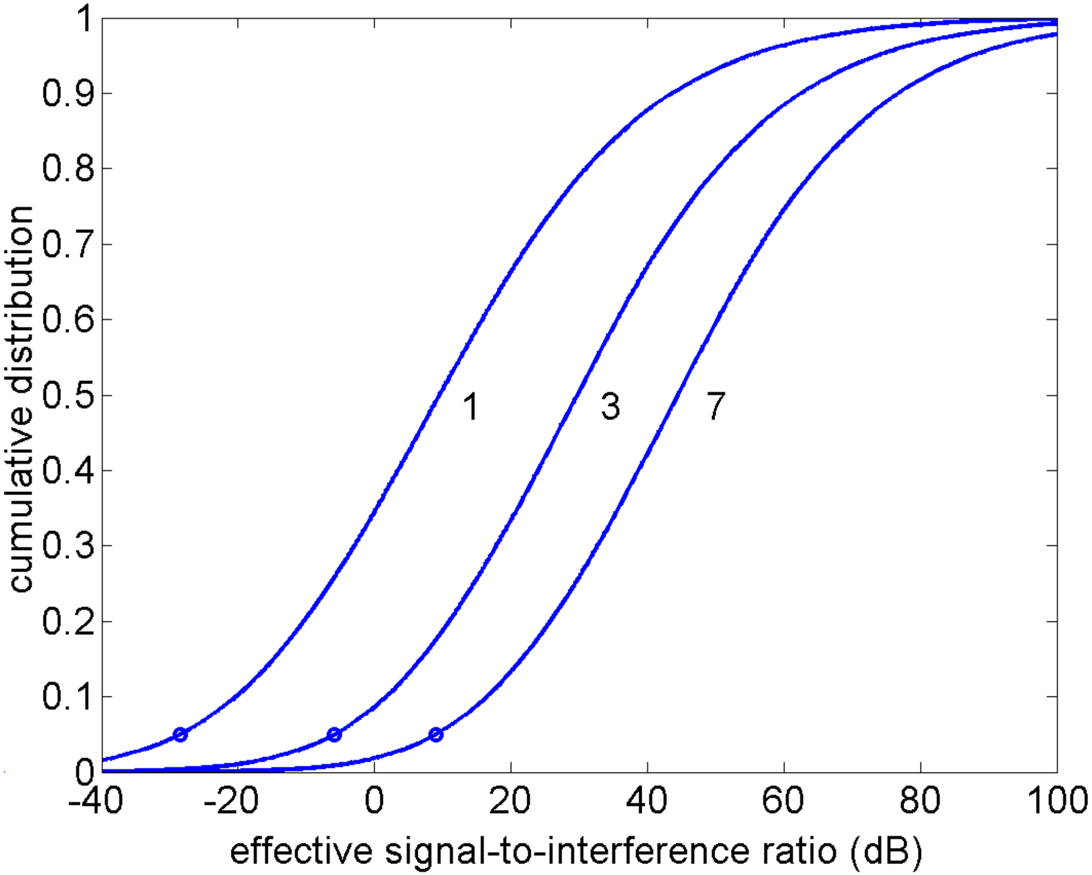
Assume Gaussian signals; SIR and capacity are random via slow fading (geometric and log-normal shadow)



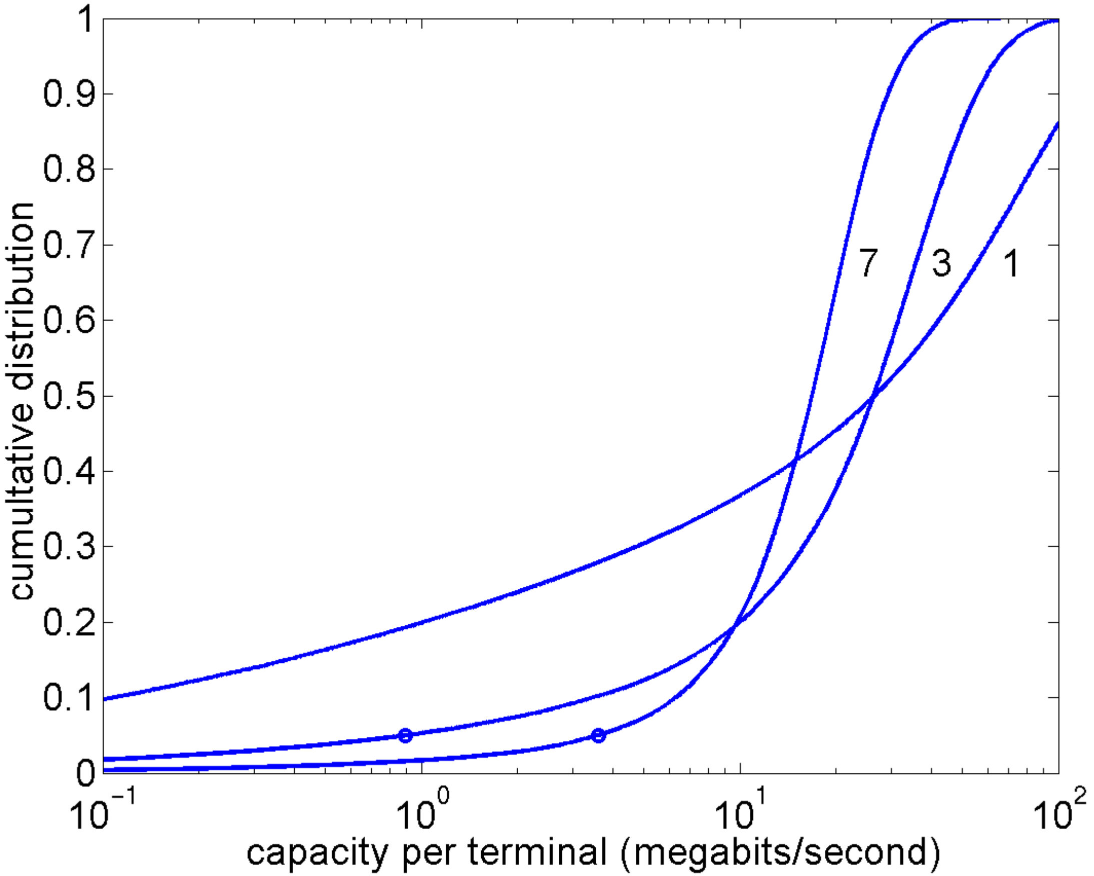
$$\mathrm{SIR}_{\mathrm{f}k} = \frac{\beta_{\ell k\ell}^{2}}{\sum\limits_{j\neq\ell} \beta_{jk\ell}^{2}}$$

$$C_{\mathrm{f}k}\ (\text{bits/s}) = \left(\frac{\text{bandwidth}}{\text{re-use factor}}\right)\left(\frac{T_{\text{data}}}{T_{\text{slot}}}\right)\left(\frac{T_{\text{usable}}}{T_{\text{symbol}}}\right)\cdot \log_2\left(1+\mathrm{SIR}_{\mathrm{f}k}\right)$$

Forward SIR: Cumulative Distribution For Re-Use Factors 1, 3, 7



Forward Capacity Per Terminal: Cumulative Distribution For 20 MHz Bandwidth



Infinitely Many Antennas: Forward-Link Capacity For 20 MHz Bandwidth, 42 Terminals per Cell, 500 μsec Slot

Interference-limited: energy-per-bit can be made arbitrarily small!

Frequency Reuse	.95-Likely SIR (dB)	.95-Likely Capacity per Terminal (Mbits/s)	Mean Capacity per Terminal (Mbits/s)	Mean Capacity per Cell (Mbits/s)
1	-29	.016	44	1800
3	-5.8	.89	28	1200
7	8.9	3.6	17	730

				Mean Capacity per Cell (Mbits/s)
LTE Advanced (>= Release 10)				74

Conclusions in the Limit of an Infinite Number of Antennas

- *Mathematically Exact*
  - **Required transmit energy per bit is arbitrarily small!**
  - Total throughput per cell is independent of cell-size
  - Number of terminals served is independent of cell-size
  - Spectral efficiency independent of bandwidth
  - Effects of uncorrelated noise and fast fading disappear
  - The *only* remaining impediment is inter-cellular interference due to *pilot contamination*
- *Approximate*
  - Number of mobiles per cell is one-half of the ratio of the coherence time to the delay-spread
  - Throughput per terminal independent of coherence time
  - Aggregate throughput per cell proportional to coherence time
  - Reverse- and forward-link throughputs nearly identical

Isolated Cell: M Antennas, K=1 Terminal

$$x = \sqrt{\rho_{\mathrm{f}}}\ \underbrace{\bar{h}}_{1\times M}\ \overbrace{\bar{s}}^{M\times 1}\ + w$$

- **Perfect CSI**

$$\bar{s} = \frac{1}{\sqrt{M}}\bar{h}^{*}q \quad \Rightarrow \quad x = \sqrt{\frac{\rho_{\mathrm{f}}}{\mathrm{M}}}\bar{h}\bar{h}^{*}q + w \approx \sqrt{\rho_{\mathrm{f}} M}\, q + w$$

$$\mathrm{SNR} \approx M\rho_{\mathrm{f}}$$

- **CSI estimated from reverse-link pilot**

$$\hat{\bar{h}} = \bar{h} + \frac{1}{\sqrt{\rho_{\mathrm{r}}}}\bar{v}, \quad \bar{s} = \frac{1}{\sqrt{M(1+1/\rho_{\mathrm{r}})}}\hat{\bar{h}}^{*}q$$

$$x = \sqrt{\frac{\rho_{\mathrm{f}}\rho_{\mathrm{r}}}{M(1+\rho_{\mathrm{r}})}}\bar{h}\left(\bar{h}^{*} + \frac{1}{\sqrt{\rho_{\mathrm{r}}}}\bar{v}^{*}\right)q + w$$

$$\mathrm{SNR} \approx \frac{M\rho_{\mathrm{f}}\rho_{\mathrm{r}}}{1+\rho_{\mathrm{f}}+\rho_{\mathrm{r}}}$$